Exotic Options I
Asian Options

→ path-dependent option

→ option on average price
XYZ has monthly inflow of €100m.

Its cost are fixed in $.

Let $X_i$ = dollar price of a euro in month $i$.

$$12 \times 100 \text{m} \rightarrow \sum_{i=1}^{12} \$100 \cdot X_i e^{r(12-i)/12}$$

$$= 100 \sum_{i=1}^{12} X_i e^{r(12-i)/12}$$

$$
\sum_{i=1}^{12} X_i = 12 \cdot \overline{X}
$$
2 kinds of Average

Arithmetic: \( A(T) = \frac{1}{N} \sum_{i=1}^{N} S_{ih} \) \( \text{like } \bar{X} \)

Geometric: \( G(T) = \left( S_{h} \cdot S_{2h} \cdot S_{3h} \cdots S_{Nh} \right)^{\frac{1}{Nh}} \) \( \text{like } \sqrt[log(x)]{e} \)
$S_{14} = 55, 72, 61, 85$

$A(t) = \frac{55 + 72 + 61 + 85}{4} = 68.250$

$G(t) = \left(55, 72, 61, 85\right)^{\frac{1}{4}} = 67.315$
Average $S_T$ or $K$

If $S_T > K$

Average price option: 
Call: $\max[0, S_T - K]$
Put: $\max[0, K - S_T]$

Average strike option: 
Call: $\max[0, S_T - G(T)]$
Put: $\max[0, G(T) - S_T]$

$2 \times 4 = 8$ types of Asian option.
XYZ could use...

Arithmetic Average price put Asian put

$$\text{Payoff} = \max(0, k - \frac{1}{12} \sum_{i=1}^{12} X_i)$$

**TABLE 14.2**

Comparison of costs for alternative hedging strategies for XYZ. The price in the second row is the sum of premiums for puts expiring after 1 month, 2 months, and so forth, out to 12 months. The first, third, and fourth row premiums are calculated assuming 1 year to maturity, and then multiplied by 12. Assumes the current exchange rate is $0.9/€, option strikes are 0.9, \( r_s = 6\% \), \( r_e = 3\% \), and dollar/euro volatility is 10%.

<table>
<thead>
<tr>
<th>Hedge Instrument</th>
<th>Premium ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Put option expiring in 1 year</td>
<td>0.2753</td>
</tr>
<tr>
<td>Strip of monthly put options</td>
<td>0.2178</td>
</tr>
<tr>
<td>Geometric average price put</td>
<td>0.1796</td>
</tr>
<tr>
<td>Arithmetic average price put</td>
<td>0.1764</td>
</tr>
</tbody>
</table>
Barrier Options

→ Set level = 'barrier'

→ If stock price hit the barrier, then the option may come into existence or go out of existence.

→ If they exist, they are same as ordinary puts and calls.

→ No more expensive than regular puts/calls.
Illustration of a barrier option where the initial stock price is $100 and the barrier is $75. At $t = 0.5$ the stock hits the barrier.
When does it 'hit' the barrier?

→ Price of stock can be manipulated by large order.

→ Barrier is defined by average over certain period of time.

→ Different firms may use somewhat different definition.
Types of Barrier Options

1. Knock-out: go out of existence if it hits barrier.
   - Down-and-out
   - Up-and-out

2. Knock-in: comes into existence if it hits barrier.
   - Down-and-in
   - Up-and-in

   - Up-rebate
   - Down-rebate
Rarity for Barrier Option

(Knock-in) + (Knock-out) = Ordinary Option

Ex.

Down-and-in Call + Down-and-out Call = Call
Compound Option

\[ \rightarrow \text{Option on Options} \]

\[ \rightarrow \text{Put/call option that has put/call option as underlying asset.} \]
Compound Option Parity

PC parity on regular options

\[ C - P = S - Ke^{-rT} \]

PC parity on compound option

\[ (\text{Call}_{x \to T}^{\frac{K}{t_1}}) - (\text{Put}_{x \to T}^{\frac{K}{t_1}}) = \text{Call}_K - xe^{-rt_1} \]
**GAP Options**

Regular Call: payoff = $S_T - K$ if $S_T > K$

Gap Call: payoff = $S_T - k_1$ if $S_T > k_2$

Diagram:

- Regular Call path
- Gap Call path
- $k_1$, $k_2$, $S_T$ lines
**Gap Call**

\[ k_1 = 90 \]
\[ k_2 = 100 \]

**Gap Put**

\[ k_1 = 90 \]
\[ k_2 = 100 \]

Required to sell
Exchange Options

(out performance option)

A pays off only if underlying asset
outperforms some other asset (benchmark)