Market-Maker

M-M makes profit by bid-ask spread.
Problem: Market-Maker is left with position determined by market demand.

- Delta hedging.

- Sold K = 40 - Call @ $2.7804

\[
\begin{align*}
\Delta &= 0.5824 \\
\Gamma &= 0.0652 \\
\Theta &= -0.0173 \\
S &= 40 \\
\sigma &= 0.3 \\
T &= \frac{91}{365} \\
\delta &= 0
\end{align*}
\]
Day 0.  Sold 40-strike Call @ $2.7804 a 100 shares,

$ 278.04

$ \Delta = -0.5824$

→ buy 58.24 shares @ $S = 40$

$= -2329.6$

$278.04 - 2329.6 = -2051.56$  

[Boxed]  

Borrow 2051.56 with 1r = 8%,

$2051.56 \cdot \frac{0.08}{365} \cdot (e^{\frac{1}{365}} - 1) = 0.45$  

overnight interest charge.
Day 2

Marking to market

\[
S = 40 \Rightarrow 40.50
\]

\[
T = \frac{90}{365} \Rightarrow \frac{90}{365}
\]

New Option price \((by\ B-S)\) = 3.0621

\[
-306.21 + 278.04 = -28.17 \quad \text{loss on written Call}
\]

58.24 shares of stock

\[
58.24 (40.50 - 40) = 29.12 \quad \text{Gain on stock}
\]

2051.51 borrowing.

\[
- (e^{0.08(365)} - 1) \times 2051.51 = -4.45 \quad \text{Interest}
\]

\[
\frac{.50}{.50} \quad \text{over high profit}
\]
Day 1  

Rebalancing the portfolio

New $\Delta = 0.6142$

$61.42 - 58.14 = 3.18$

Additional stock needed

$40.50 \times (3.18) = \boxed{128.19}$

Additional equity investment needed

Borrowing Capacity

$61.42 (\$40.50) \text{ stock}$

Day 0  Int need

$2051.56 + .45 + 128.19$

= 2180.10

.50 difference

\$ cash in

\$2181.30

borrow this anyway
Day 2

Marking to Market

\[ S = 40.50 \rightarrow 39.25 \]

\[ T = \frac{90}{365} \rightarrow \frac{89}{365} \]

New option price = 2.3282

\[-232.82 + 306.21 = 73.39\] Gain on written call

61.42 shares of stock

\[ 61.42 \left( 39.25 - 40.50 \right) = -76.78 \] Loss on stock

Borrowing 2181.30

\[ \left( e^{0.02 \cdot \left( \frac{1}{365} \right)} - 1 \right) \times 2181.30 = -48 \] Interest

\[ - \$3.87 \] overnight profit
<table>
<thead>
<tr>
<th></th>
<th>Day</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Stock ($)</td>
<td>40.00</td>
</tr>
<tr>
<td>Call ($)</td>
<td>278.04</td>
</tr>
<tr>
<td>Option delta</td>
<td>0.5824</td>
</tr>
<tr>
<td>Investment ($)</td>
<td>2,051.58</td>
</tr>
<tr>
<td>Interest ($)</td>
<td>-0.45</td>
</tr>
<tr>
<td>Capital gain ($)</td>
<td>0.95</td>
</tr>
<tr>
<td>Daily profit ($)</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Daily profit calculation over 5 days for a market-maker who delta-hedges.
Self-financing portfolio

- Portfolio that does not require additional cash.
Binomial pricing

Payoff

Call

$\Delta S + B$

$A, K, B$

Payoff

written Call

$A, B$

Payoff

Combined

Profit

$A, K, B$
Self-financing portfolio

Overnight profit = 0 if

Stock price move by ± σ.

If stock move around by ± σ every day,
delta-hedged portfolio will be
self-financing.

delta-neutral portfolio: portfolio that is delta-hedged
Delta - Gamma Approximation

\[ \Delta : \text{change in } C \text{ when } S \uparrow \$1 \]

\[ T : \text{change in } \Delta \text{ when } S \uparrow \$1 \]

\[ S = 40 \rightarrow 40.75 \]

\[ C = 2.804 \rightarrow 3.2352 \]

\[ \Delta = 0.5824 \]

\[ T = 0.0652 \]

by \( \Delta \) only

\[ \frac{C(40.15) - C(40)}{1} = 0.75 \times 0.0584 = 0.04368 \]

\[ C(40.75) = C(40) + 0.4368 \]

\[ \approx \]

\[ 3.2172 \]
Taylor Series Approximation

\[ f(x) = f(a) + f'(a)(x-a) + \frac{1}{2} f''(a)(x-a)^2 + \cdots \]

\[ C(\text{new } s) = f(s) + \Delta (\text{change in } s) \quad \text{1st order} \]

\[ C(\text{new } s) = f(s) + \Delta (\text{change in } s) + \frac{1}{2} \Delta^2 (\text{change in } s)^2 \quad \text{2nd order} \]
Delta - Gamma Approximation

\[ C(S_{t+h}) = f(S_t) + \Delta (S_{t+h} - S_t) + \frac{1}{2} \sigma^2 (S_{t+h} - S_t)^2 \]

\[ C(40.75) \approx C(40) + (0.5824)(0.75) + \frac{1}{2} (0.0652)(0.75)^2 \]

\[ \approx 3.2355 \]

true value

\[ C(40,75) = 3.2352 \]
FIGURE 13.3
Delta- and delta-gamma approximations of option price. The true option price is represented by the bold line, and approximations by dashed lines.
Use $\Theta$ as well

\[ C(\text{new } S \text{ tomorrow}) = C(\rightarrow) + \Delta (\text{change in } S) + \frac{1}{2} T (\text{change in } S)^2 + \Theta(1) \uparrow \text{1 day}. \]
Predicted option price over a period of 1 day, assuming stock price move of $0.75, using equation (13.6). Assumes that $\sigma = 0.3$, $r = 0.08$, $T - t = 91$ days, and $\delta = 0$, and the initial stock price is $40.

<table>
<thead>
<tr>
<th>Starting Price</th>
<th>$\epsilon \Lambda$</th>
<th>$\frac{1}{2} \epsilon^2 T$</th>
<th>$\theta h$</th>
<th>Option Price 1 Day Later ($h = 1$ day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{t+h} = 40.75$</td>
<td>2.7804</td>
<td>0.4368</td>
<td>0.0183</td>
<td>$-0.0173$</td>
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