Swaps
Swap
- like forwards, but in series.

Example: IP Inc. - need to buy oil in 1yr and 2yr

- Forward price $20 in 1yr
- $21 in 2yr
- 100,000 barrels
- 6% risk-free rate for 1yr
- 6.5% for 2yr

If we enter long in 2 forwards, we need to come up with cash:

\[
\begin{align*}
\text{2M in 1yr.} \\
\text{2.1M in 2yr.}
\end{align*}
\]
\[ PV = 2m \left( \frac{1}{1.06} \right) + 2.1m \left( \frac{1}{1.65} \right)^2 \]

\[ = \$ 3,783,000 \]

Prepaid Swap: Pay $3,783,000 today, get 100,000 barrels in 1 yr and in 2 yr.
Swap

Pay $X$ in 1yr for $100,000$ barrel of oil.

$X$ in 2yr for $100,000$ barrel of oil.

where

$$\frac{X}{1.06} + \frac{X}{1.06^2} = \frac{20}{1.06} + \frac{21}{1.06^2} = 37.38,300$$

$X = \$20.483 \times 100,000$

↑ 2yr swap price.

This is physical settlement.
Financial Settlement.

Pay $20.483 \times 100,000 \text{ in } 1\text{yr + 2yr}.

Seller of this swap compensates the difference.

<table>
<thead>
<tr>
<th>1yr</th>
<th>2yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot Price</td>
<td>$20</td>
</tr>
<tr>
<td>Spot Price</td>
<td>$21</td>
</tr>
</tbody>
</table>

\[ 6 \text{ notional amount} \]

\[ \text{\$1 - \$0.5 \times 100,000} \]
Market Value of a Swap

\[ \begin{align*}
1y & \quad 2y \\
\text{Agreed to pay } & \quad 20.483 \quad 20.483 \text{ instead of } \\
& \quad 20 \quad 21
\end{align*} \]

Value of Swap = 0, until 1st swap is struck.

What if, after the agreement, spot price of oil rises, and forwards are available at

\[ \begin{align*}
& \quad 22 \quad 1y \\
& \quad 23 \quad 2y
\end{align*} \]
New swap

\[
\frac{22}{1.06} + \frac{23}{1.065^2} = \frac{X}{1.06} + \frac{X}{1.065^2}
\]

\[X = 22.483\]

Difference

\[
\frac{2}{1.06} + \frac{2}{1.065^2} = 3.650
\]

Value of old swap = 3.650
Interest Rate Swaps

XYZ corp. $200m debt at floating-rate. LIBOR

want fixed rate.

1. Refinance with fixed-rate. (transaction cost)

2. May forward rate agreements. (Cost will vary each year)

3. Interest rate swaps.
5 Ways to Present Risk-Free Rates

<table>
<thead>
<tr>
<th>Years</th>
<th>Zero Coupon Yield</th>
<th>Zero Coupon Yield</th>
<th>1yr Implied Forward Rate</th>
<th>Par Coupon (annual)</th>
<th>Continuously Compounded</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6%</td>
<td>.943396</td>
<td>6%</td>
<td>6%</td>
<td>5.22689 %</td>
</tr>
<tr>
<td>2</td>
<td>6.5%</td>
<td>.881659</td>
<td>7.0236%</td>
<td>6.48423 %</td>
<td>6.29788 %</td>
</tr>
<tr>
<td>3</td>
<td>7%</td>
<td>.816298</td>
<td>8.00705%</td>
<td>6.95783 %</td>
<td>6.76586 %</td>
</tr>
</tbody>
</table>
1. **Zero-Coupon Yield**
   - **Yield Curve**
   - **Spot Rate**

2. **Zero-Coupon Bond Price**
   - 
   - **PV of $1 in year n**

3. **1yr Implied Forward Rate**

<table>
<thead>
<tr>
<th>Year n</th>
<th>Zero-Coupon Yield</th>
<th>Bond Price</th>
<th>1yr Implied Forward Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.06</td>
<td>(\frac{1}{1.06})</td>
<td>1.06</td>
</tr>
<tr>
<td>2</td>
<td>1.065</td>
<td>(\frac{1}{1.065})</td>
<td>(\frac{1.065^2}{1.06})</td>
</tr>
<tr>
<td>3</td>
<td>1.07</td>
<td>(\frac{1}{1.07})</td>
<td>(\frac{1.07^3}{1.065^2})</td>
</tr>
</tbody>
</table>
Bond with PV = 1  
Face = 1  
Coupon = C

\[ 1 = C \left( \frac{1}{1.06} \right) + C \left( \frac{1}{1.06^2} \right) \]  
\[ 1 = C \left( \frac{1}{1.06} \right) + C \left( \frac{1}{1.065^2} \right) + C \left( \frac{1}{1.065^3} \right) \]  
\[ 1 = C \left( \frac{1}{1.06} \right) + C \left( \frac{1}{1.065^2} \right) + C \left( \frac{1}{1.07^2} \right) + C \left( \frac{1}{1.07^3} \right) \]  
\[\text{Continuously compounded}\]  
\[\ln(1.06)\]  
\[\ln(1.065)\]  
\[\ln(1.07)\]
Interest Rate Swaps

1yr Forward rate - we can lock in today.

\[
\frac{R - 6\%}{1.06} - \frac{R - 7.0024\%}{1.065^2} - \frac{R - 8.00725\%}{1.07^3} = 0
\]

\[
R = 6.9548\%
\]

Swap rate = Par coupon rate

Lock in rate for 3 yrs.
Swap rate = your bond coupon

Zero-coupon bond yield

- 6.00
- 6.50
- 7.00

One-year implied Euro

\[
\frac{(6.50)^2}{6.00} = 7.0236
\]

\[
\frac{1.07^3}{(1.05^2)} = 8.0905
\]

\[
l = \frac{c}{1.06} + \frac{c}{1.08^2} + \frac{1}{1.06^2} \quad C = 0.06484
\]

\[
l = \frac{c}{1.06} + \frac{c}{1.08^2} + \frac{c}{1.06^2} \quad C = 0.06954
\]

Pay:

\[R - 6\% \quad R - v_2 \quad R - v_3\]

Counter party:

\[R - v_2 - R \quad R - F_1 \quad R - F_2 \quad R_v - F_3 \quad \text{negliged.}\]

You