Comment on "A Note on the Two-Spring Tomlinson Model"

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A recent note by Lu et al presented a theoretical study of two-spring Tomlinson model. In this comment we argue that the conclusion of the note can be understood in a simple manner, moreover we point out the discussion of the criterion for the onset of stick-slip in the note is unnecessarily complicated and potentially misleading.

A recent *Tribology Letters* Note reported an analytical study of the two-spring, two-mass Tomlinson model under quasistatic conditions [1]. The main points of that article were (i) the quasistatic solution of the two-spring Tomlinson model is equivalent to the one-spring model provided the effective spring stiffness is correctly defined, and (ii) a criterion for the onset of stick-slip expressed in terms of explicitly defined tip and cantilever stiffness is the most convenient way to study the effects of tip flexibility. In this Comment we show that the first of these points is readily apparent from a simple analysis and the second is supported by results presented in an unnecessarily complicated and potentially misleading manner.

We are in agreement with the authors' first conclusion that the quasistatic solution of two-spring model is equivalent to that of the one-spring model (Eq. 20 of [1]). However, this conclusion is straight forward since under quasistatic conditions inertia can be neglected. Regardless of how many masses the system has (consider the n-spring, n-mass system illustrated in Figure 1), it simplifies to a one-spring model with an effective stiffness k_{eff} satisfying

$$\frac{1}{k_{eff}} = \sum_{i=1}^{n} \frac{1}{k_n}. \tag{1}$$

The quasistatic two-spring, two-mass system with tip and cantilever stiffness k and K, respectively, is simply a special case with $\frac{1}{k_{eff}} = \frac{1}{k} + \frac{1}{K}$.

Transitions between slip regimes for this system have been systematically investigated by Medyanik *et al.* [2] and can be determined by $\eta = \frac{2\pi^2 U_0}{a^2 k_{eff}}$; where $\eta < 1$ corresponds to smooth sliding, $1 < \eta < 4.6$ to single slip, $4.6 < \eta < 7.8$ to double slip, and so on. In the Note, the authors introduce a new parameter, $\eta_2 = \frac{2\pi^2 U_o}{a^2 k}$ (Eq. 10 of [1]), and conclude that the criterion for the onset of stick-slip motion is (Eq. 13 of [1])

$$\eta_2 > 1 - \frac{k}{K+k}. \tag{2}$$

However, this equation is mathematically equivalent to

$$\eta > 1 \tag{3}$$

where $\eta = \frac{2\pi^2 U_o}{a^2 k_{eff}}$ and $k_{eff} = \frac{Kk}{K+k}$. Therefore, the introduction of a new parameter η_2 in the Note and the corresponding discussion in Section 4 (Comparison of the Two Models) makes the analysis unnecessarily complicated.

Another consequence of the definition of η_2 is the misleading discussion in Section 5 (Equivalent Forms of the Criteria for Predicting Stick-Slip Motion). As stated in the Note, "Figure 2 shows the general behavior of η_2 with k/K, indicating the values of k/K leading to the onset of stick-slip motion." However, Figure 2 of [1] gives the false impression that k and K are asymmetrical. Further, this figure incorrectly suggests that the larger the magnitude of k/K, the more likely stick-slip will occur. In fact, K (cantilever stiffness) and k (tip stiffness) are mathematically equivalent or physically symmetric in determining the onset of stick-slip for a *quasistatic* system. The primary source of the confusion created by Figure 2 is the definition of the axes, $\eta_2 = \frac{2\pi^2 U_o}{a^2 k}$ and k/K, both of which contain the variable k. Since the x and y axes are not independent, the resultant plot cannot convey the intended physical meaning. Indeed, all of the discussion contained in Section 5 can be summarized by $\eta > 1$, or equivalently $\frac{2\pi^2 U_o}{a^2} > \frac{Kk}{K+k}$ in which the effect of tip flexibility in determining transitions to stick-slip is explicitly clear.

Overall, the contents of the Note can be conveyed by two simple equations: $k_{eff} = \frac{kK}{k+K}$, the effective stiffness of quasistatic springs in series; and $\frac{2\pi^2 U_o}{a^2 k_{eff}} > 1$, the criteria for the onset of stick-slip. These expressions are already known, and they describe the relationship between one-spring and multiple-spring models as well as the effect of tip flexibility in determining transitions to stick-slip, simply and without defining new variables.

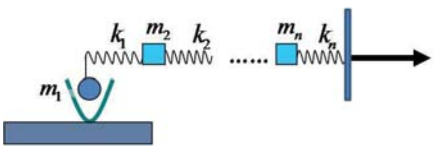

FIG. 1: Illustration of the multiple-spring Tomlinson model.

References

[1] Lu, P., Loke, Y., Tang, X., Kushvaha, S. and OShea, S.: A note on the two-spring Tomlinson model. Tribol. Lett. **43**, 73–76 (2011)

[2] Medyanik, S., Liu, W., Sung, I. and Carpick, R.: Predictions and observations of multiple slip modes in atomic-scale friction. Phys. Rev. Lett. **97**, 136106 (2006)