Strain Measurement – Worksheet for data analysis

This worksheet is designed to help you write the report and verify that you have sufficient data to complete the analysis. Remember to show your work in the notebook. If you use software to calculate the regression coefficients, cut and paste your work into the notebook. Pay attention to units.

EQUATIONS:
The surface stress in a beam in bending is determined from the beam geometry and the load

\[ \sigma_{\text{axial}} = \frac{M}{I} \frac{h}{2}, \quad I = \frac{bh^3}{12}, \quad M = F \cdot l = m \cdot g \cdot l \]

Combining the above equations yields for the axial surface stress

\[ \sigma_{\text{axial}} = \frac{12F}{bh^3} \frac{l}{2} = \frac{6Fl}{bh^2} \]

where \( m \) is the mass of the added weights and \( g \) is standard gravity.

The strain is determined by the voltage measured from the strain gage bridge amplifier

\[ V_e = K \cdot \varepsilon_{\text{axial}} \]

The modulus of elasticity is determined from

\[ E = \frac{\sigma_{\text{axial}}}{\varepsilon_{\text{axial}}} \]

Combining these equations yields the dependence of the E-modulus on test variables

\[ E = \frac{6mgl}{bh^2} \frac{K}{V_e} \]

QUESTIONS:

1. Plot the calculated axial stress (y-axis) vs. the axial strain (given by the mean of the data acquisition measurements) for all of your weight trials both increasing and decreasing weight. Fit the data with a linear least-squares regression. Show the regression line and the individual data points on your plot. Determine the precision interval for the line at 95% probability.

   Use Excel, MATLAB or similar tool to generate this graph. Use the equations given in lecture 4, page 4 and 5 to calculate the regression line. Refer to the example on page 5 in the lecture notes. Recall that \( a_1 \) is the slope (=E-modul). Discuss the meaning of \( a_0 \) in your report. Write an equation of the form:

   \[ \sigma = a_0 + a_1 \varepsilon \pm t_{y.95\%} S_{\sigma \varepsilon} \]

2. The slope of the line (the \( a_1 \) coefficient from the fit) is your calculated modulus of elasticity. Evaluate the 95% precision interval of the measured modulus. This may be accomplished by finding the standard deviation of the slope of the fit, \( S_{a1} \), and the appropriate \( t_{95} \) factor.

   Recall that \( S_{a1} \) is found from
3. Using the modulus of elasticity from question 1 above, calculate the axial strain from the axial stress at the location of the transverse strain gages for each of the weight trials. Plot the transverse strain given by the mean of the data acquisition measurements vs. the axial strain calculated above. Fit the data with a linear least-squares regression and evaluate the 95% precision interval. The slope of the line is your Poisson's ratio. Use a similar procedure as in question 2 to evaluate the precision of your predicted value.

What is the stress at the location of the transverse strain gages? Why is it different from the stress at the axial strain gages? Did you record sufficient information to determine the location of the transverse strain gages? What is the uncertainty associated with the strain gage location. For this question you calculate the axial strain from the axial stress and the estimated E-modul from above. The transverse strain is measured via the strain gage.

\[ \nu_p = - \frac{\varepsilon_{\text{transverse}}}{\varepsilon_{\text{axial}}} \]

Again a plot can be generated based on axial and transverse strains. A linear regression yields the best fit and slope, which is the Poisson Ratio. Determine the confidence interval as before.

4. To find the total uncertainty in the modulus, you must determine sources of bias error. The calculated stress will have errors from uncertainties in the specimen width and height, the position of the strain gages, and the force applied by the mass, while the strain measurement will be affected by the accuracy of the BAM calibration (including drift over the term of the experiment) and the accuracy and resolution of the data acquisition. There may also be boundary conditions or other factors that will bias the result. Use the propagation of error technique to combine these individual uncertainties into a single bias uncertainty for modulus. Do this by writing the equation for modulus in terms of the parameters above and calculating or measuring the derivatives, paying close attention to units.

Since the beam deforms during the load application, the angle between the load direction and the beam axis deviates from 90 degrees. This can be included by considering a load angle \( \theta \). Determine this angle by adding 10lbs of weight to the beam and measuring the deviation from 90 deg.

Considering the load angle the equation for the E-modul becomes

\[ E = 6 \frac{mgl}{bh^2} \frac{K}{V_z} \cos \theta \]

Recall from the lecture that the relative uncertainty \( e_i \), associated with each independent variable \( x_i \), is
The total relative uncertainty $e_y$ is

$$e_y = \sqrt{e_1^2 + e_2^2 + \ldots + e_i^2}$$

Using these equations determine the uncertainty for determining the modulus of elasticity $E$. Discuss with your partner the uncertainty associated with each parameter and review, whether you have sufficient data to estimate its value. Discuss with your TA if necessary:

- Weight Mass $m$
- Gravity $g$
- Length $l$
- Width $b$
- Height $h$
- Strain gage factor $K$
- Voltage $V_e$
- Angle $\theta$
- Other sources of error?

Calculate the uncertainty by following the example of the Pitot Static tube in the lecture.

5. **Estimate the total uncertainty in the modulus of elasticity for your test specimen by combining your precision interval in modulus from step 1 with the bias error from step 4. Compare the result to published values for your specimen material.**

Recall that the total uncertainty is obtained by combining bias error and precision error using the RSS method. Write an equation, which shows the 95% confidence interval for the E-modul using the bias uncertainty from step 4 and the precision interval from step 2.

$$E_{95\%\text{confidence}} = E_{\text{fit}} \pm \sqrt{E_{\text{uncertainty}}^2 + (t_{0.95,\text{S}_{\text{a}1}})^2}$$

**CONCLUSIONS:**

The conclusions should state the modulus of elasticity and Poisson's ratio found. Include the total error estimate for the modulus and the precision error for Poisson's ratio. The comparison to reference values (cite the reference used) should also be presented along with discussion of possible differences between the measured and stated values. Is the theory for calculated stresses and strains (rather than measured ones) exactly correct for the experiment? Does the physical limitations of the experiment or boundary conditions effect the accuracy of the measured values? Recommend the most effective ways to improve the experiment, using both your error analysis and the discussion to justify your suggestions.