Strain Measurement

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I. Objective

During this laboratory experiment, the strain in a loaded cantilever beam will be measured in both the axial and transverse directions using strain gages and a Wheatstone Bridge Circuit. From the measured strain and the calculated stress the Young’s modulus and the Poisson's ratio of the beam material will be determined. These values will be compared to the material properties of the beam as reported in references.

An estimate of the uncertainty in the measurements will be completed using a statistical analysis, including an estimation of the source contribution of the errors.

The independent variable in this experiment is the applied load. The dependent variables are the strains on the surface of the cantilever beam at the strain gage locations.

II. Apparatus

1. a metal cantilever beam mounted in a clamp support
2. a C clamp to hold the base to the table,
3. two 120.5Ω axially mounted strain gages (G_f = 2.09 ± 5%),
4. two 120.5Ω transversely mounted strain gages (G_f = 2.09 ± 5%),
5. a ten pound set of weights with pan and hanger,
6. NI9219 DAQ Card with built in Wheatstone Bridge and Amplifier,
7. a Digital Multimeter (DMM) with 10 MΩ input impedance (optional)
8. Ruler, micrometer, caliper
9. Weight Scale  

III. Principles and Background  

Force measurement is an important part of many measurement systems. Pressure, drag, torque, and other parameters may be indirectly measured by measuring the forces applied to a device. One of the best ways to measure force is also an indirect method: measurement of the strain caused when an applied force induces stress in an object and thus deformation. This experiment relates the stress to the strain in a cantilever beam loaded with a point load.

The definition of stress is Force/Area. For a beam in tension, stress is simply related to axial force by

$$\sigma_{axial} = \frac{F_{axial}}{A_{axial\,cross\,-\,section}}$$

In many materials, stress relates linearly to strain over an elastic range with Hooke's law. For a simple one-dimensional stress state (non-zero stress only acting in one direction), it is given by the equation

$$\epsilon_{axial} = \frac{\sigma_{axial}}{E}$$

where E is the material modulus of elasticity (or Young’s modulus). This linear relationship ends at high stresses when the yield point is reached and plastic deformation occurs. For materials, where Hooke’s law applies, we can measure strain to find stress through a linear equation.

Strain is a physical second-order tensor, (i.e. it has nine components, because of the symmetry of the tensor six of these components are unique: 3 normal strains and 3 shear strains), which describes the distortion of an elastic medium at a given point. On a free surface, the strain at a point may be described with three components (2 normal strains and 1 shear strain). These three components may be two perpendicular normal strains and the shear strain between these two directions. If the directions of principal strain are known, these directions and the two values of principal normal strain are sufficient to define the state of strain at that surface point. If these directions are not known, a strain rosette must be used to measure the state of strain at that point.

Extensional strain is defined as the change in length divided by the original length.

$$\epsilon = \frac{\Delta L}{L}$$

Strain such as stress is a tensor with three normal strains and three shear strains defining the local deformation. Strain varies throughout an elastic object. When measuring strain, the strain is an average value over the original length L. Therefore the smaller the original length of the specimen, the more localized the measurement of the strain (Note: L here is any arbitrary length over which strain is measured, not to be confused with L, the beam length).

Transverse strain is defined as the strain transverse (normal) to the direction of reference. If the direction of reference is the axial beam direction, the transverse strain is normal to the axial direction. When a beam is subjected to positive axial strain, the beam cross-section narrows, thus
giving rise to a negative transverse strain. For a one-dimensional stress state (stress only occurs in one direction, it is zero in the other directions) the relation of axial to transverse strain is defined as the Poisson’s ratio. It is a material property and defines the amount of “contraction” of the specimen in the transverse direction under uniaxial stress. It is defined by the equation

$$V_p = -\frac{\varepsilon_{\text{transverse}}}{\varepsilon_{\text{axial}}}$$

Because strains are changes in dimension, normal surface strains are easily measured quantities. Many methods and devices are available to measure localized displacements. Other quantities such as stress, force, torque and pressure are often determined by measuring strain in a controlled device (transducer), and relating the strain to the quantity of interest.

**Strain in Cantilever Beams**

In this lab we’ll be measuring strain on the surface of a cantilever beam. The applied load at the beam tip creates a bending moment $M$, which creates a normal stress and strain distribution in the beam cross section. The surface strain can be measured at the top or bottom surface of the beam by measuring the local change in length.

For the bending deformation of the cantilever beam shown, each cross sectional plane will undergo a rotation $\kappa$, which will produce strains and stresses in the cross section, growing linearly outward from the neutral axis of bending ($\varepsilon_x = \kappa y$). It can be shown that the axial stress, $\sigma_{\text{axial}}$, at the upper and lower surface ($y = h/2$) is given by:

$$\sigma_{\text{axial}} = \frac{M h}{I}$$

where $M$ is the bending moment, $I$ the moment of inertia and $h$ the thickness of the beam. We calculate the second moment of area, $I$ for a rectangular beam cross section from

$$I = \frac{bh^3}{12}$$

The maximum bending moment, $M$, at any position, $d = L-x$, relative to the location of the force, $F$ is

$$M = F \cdot d$$

The axial strain, $\varepsilon_{\text{axial}}$, at the upper and lower surface is related to the stress through the material behavior, which is assumed to follow Hooke’s law.

$$\varepsilon_{\text{axial}} = \frac{\sigma_{\text{axial}}}{E}$$
With these equations stress and strain can be determined from beam dimensions, material and load.

**Metal Wire/Foil Strain Gage**

Wire is a convenient way to measure strain. When strained, a wire will increase in length and thin in cross-section. This causes an increase in resistance of the wire given a constant material resistivity $\rho_e$.

$$R_{wire} = \frac{\rho_e L}{A_{wire}}$$

This effect is the basis for strain gages. Resistance-type strain gages may be constructed of either thin metal wire or a thin metal foil. Metal foil gages are the most common. They can be produced inexpensively in a variety of patterns by an etching process similar to that used in electronics manufacturing. The gages are available in lengths ranging from 0.008" to 4" and can be as inexpensive as $1 apiece. Generally, larger gages can have higher gage resistance and can operate at higher working voltages, giving better sensitivity. They also average strain over a large area, which may be a disadvantage, if localized strain measurements are needed. In contrast, smaller gages average out the strain over shorter lengths, thus providing a more precise value of the local strain. The nominal gage resistances are standardized to 120.5 Ohms, 350.5 Ohms, and 1000.5 Ohms.

The etched foil grid is very flexible and very thin. These properties minimize its influence on the object to which it is bonded. The metal foil is cemented to a thin, flexible sheet of plastic that acts as a base for the gage and provides electrical insulation between the metal gage and the object on which the gage is bonded. The strain is transferred from the specimen to the gage through the adhesive; therefore, extreme care must be taken in proper mounting. Lead wires are soldered to two tabs on the gage and extended to a remote location where the resistance measurement circuitry is located. A simple single element gage pattern is shown in the figure above. The location of the top strain gages is shown in the image to the right. The two strain gages are oriented to measure axial and transverse strain.

**Gage Factor**

The gage factor, $G_F$, relates a unit change in length (i.e., strain) to a unit change in gage resistance. This can be expressed as:

$$\frac{\Delta R}{R} = G_F \frac{\Delta L}{L} = G_F \varepsilon$$

where $R$ is the nominal resistance of the gage, 120.5 Ohms for the gages in this lab, and $\Delta R$ is the change in resistance due to the strain.
The gage factor is a function of the gage material only, and it is not affected by the gage configuration and orientation. The gage factor is supplied by the manufacturer, and typically has a value of around 2.0 for most metal strain gages. The gages used in this lab have a nominal gage factor of $G_F = 2.09 \pm 5\%$.

A resistance-type strain gage measures strain by converting a change in length of the thin foil conductive path to a slight change in electrical resistance. The change in resistance is proportional to the change in length of the gage as shown above. Signal conditioning is needed to convert the very small change in gage resistance into a measurable change in voltage.

**Wheatstone Bridge Circuit**

One circuit that converts resistance to voltage is the Wheatstone bridge. Wheatstone bridge circuits are based on the principle of comparing the voltage drops across parallel resistance “legs”. From Kirchhoff’s laws the output voltage to input voltage ratio is found to be

$$\frac{E_0}{E_i} = \frac{R_1}{R_1 + R_3} \frac{R_4}{R_2 + R_4}$$

If the resistances in the bridge satisfy the “bridge balance” equation

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

then the bridge output voltage $E_0$ is zero. If even a small imbalance exists between the resistance ratios, the bridge generates a voltage output.

**Wheatstone Half-Bridge Circuit**

By replacing some of the resistors in the bridge by strain gages, a strain measurement circuit is obtained. There are several bridge circuits that distinguish themselves by the number of sensing resistors used. In the lab a half-bridge circuit is used as shown on the left. It uses two strain gauges on opposing arms of the bridge. By placing the strain gauges at the top and bottom of the beam, the strain gauge measure the tension and compression strain on the beam on the opposing arms.

Applying the general bridge equation to the half-bridge circuit yields an equation that relates the output voltage to the applied strain and gage factor (for a detailed derivation, see the lecture notes). Note that the actual values of the resistors drop out of the equation.

$$\frac{E_0}{E_i} = \frac{G_F}{2} \varepsilon$$
Wiring Diagram for the Wheatstone Half-Bridge Circuit on the NI9219 board

The measurements lab uses the NI9219 module to perform strain gauge measurements. This module is designed to support multiple Wheatstone bridge circuits on its 4 input channels. Each channel has a 24 bit A/D converter. It can also supply the excitation voltage for the bridge circuit.

The connector and wiring diagram is shown below.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Pin</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>C</td>
<td>EX+/HI* Positive excitation or input signal</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>HI Positive input signal</td>
</tr>
<tr>
<td>5</td>
<td>D</td>
<td>EX−/LO* Negative excitation or input signal</td>
</tr>
</tbody>
</table>

When connecting the strain gage wires to the DAQ, connection B has two leads, one from each strain gauge.
Temperature Compensation

There are two ways that temperature can affect a strain gage measurement. First, the resistance of the metal foil is a function of temperature, as well as strain, or elongation. Thus, if the temperature changes, the resistance of the gage will change. Second, if the strain gage and the object it is attached to have different thermal coefficients of expansion, then a change in temperature will actually induce unwanted strain in the gage. If only a single gage is in use, as in the quarter-bridge configuration, this change in resistance will result in an error in the measurement of the strain of the object. Note that any time a current is passed through the gage power is dissipated and can potentially heat up the gage. This problem is minimized by keeping the gage resistance high and therefore keeping current flow as low as possible.

In order to distinguish resistance changes due to strain from those due to temperature change, an additional compensating gage is used in a half-bridge configuration. The compensating gage is an identical gage that is placed on the same type of material and in the same thermal environment as the original gage. The compensating gage is then placed in the Wheatstone bridge circuit on the opposite side as the primary gage, as shown in the half-bridge configuration. Temperature changes will have an identical effect on the resistances of both gages. In the half-bridge configuration, the resistance changes will have no net effect, since both resistors are changing by the same amount thus resulting in no net change in the voltage output from the bridge.
IV. Procedure

Overview

A cantilever beam with an overhung weight pan is to be used to produce known strains (axial and transverse) at two gage locations. The test setup is illustrated on the right. In this setup, four gages are bonded onto the beam at two locations. Axial gages are placed at the top and bottom of the beam at one location while transverse gages are at the top and bottom of the beam in a second location. By clamping the base of the cantilever to the edge of the laboratory table and hanging different weights on the hanger, a series of known stresses can be generated in the beam. The associated strains can be measured with the strain gages.

In the first part of the experiment the axial half bridge is to be used to measure the axial strain in the beam at the gage location. This is done for a series of ascending and descending weights. From this data, a plot of axial strain versus axial stress can be drawn for this location on the beam. This information can then be used to calculate the elastic modulus of the beam's material.

In the second part of the experiment the transverse half bridge is to be used to measure the transverse strain in the beam at the gage location, again for a series of ascending and descending weights. One can combine this information, and the relationship between axial strain and axial stress, to develop a curve of transverse strain versus axial strain at this location in the beam and thereby determine Poisson's ratio for the material of the beam.

Finally, we'll try to ascertain and quantify different sources of error in the measurements and perform analyses on the data to determine the uncertainty. This uncertainty will result from precision and bias errors in the parameters used to calculate modulus, as well as from the boundary conditions of the experiment.

HANDLE THE STRAIN GAGES AND WIRES CAREFULLY!
DO NOT CONNECT WIRES UNTIL THE INSTRUCTIONS TELL YOU TO!

The procedure for the experiment is as follows:

1. Assemble/Inspect Beam for Axial Strain Measurement
   a. Look at the strain gages on each side of the beam. How are they oriented and wired? How many are there? Sketch your observations.
   b. How does the color of the wires and plugs relate to the strain gage orientation? Identify the double wire lead plugs. Why is there a double wire lead? Compare the wiring to the figures describing the half-bridge.
   c. Verify that the beam is clamped in the base perpendicular to the edge with the weight-pan eyebolt located 6 1/2 inches from the edge of the base.
d. Measure and record the cross sectional dimensions of the beam and the distances from the weight pan eyebolt to the centers of the four gage locations. Use the most accurate tool available for this purpose. Record the resolution of the tool.

e. Verify and record the location of the gages on the bottom of the beam. Note if the locations are not identical.

f. Record the material composition of the beam (ask your TA if you are not sure).

g. Clamp the beam to the table using a C-clamp.

2. Connect axial strain gage wires to the NI9219 DAQ Card using the half bridge configuration

a. Connect the strain gage wires for the axial strain gauges first (red wires). Which ones go to B, C, and D respectively? What does B, C, and D represent? Look back at the wiring diagram, if you are not sure.

b. Launch LabVIEW and create a new Blank VI.

c. In the block diagram add a DAQ Assistant.

d. Select acquire signal → analog input → strain.

e. Choose NI 9219 module and select analog input channel 0.

f. In strain configuration choose half bridge II. Why is this chosen? Configure the strain setup with the correct values for the strain gauges. The recommended excitation voltage provided by the board is 2.5V. Set the Gage Factor as 2.09 and the Gage Resistance as 120.5. Choose a continuous sampling rate of 100Hz and 100 samples. You can change this setting later.


3. Offset Nulling (Balancing the Unloaded Bridge)

When you wire a Wheatstone bridge based on strain gages, the bridge probably will not output exactly 0 V when no strain is applied. Slight variations in resistance among the bridge legs generate some nonzero initial offset voltage. A system can handle this initial offset voltage using software compensation.

a. Add the weigh pan to the over hung beam. This will be the “zero load” condition.

b. Select the Device tab. On this tab, you also have the option of picking a rate at which the Wheatstone bridge circuit reading is updated. Use the High Speed timing mode, which refreshes the reading from the bridge at the highest possible rate (see the NI9219 documentation for more information on Timing Modes).

c. Click Strain Calibration to launch the Strain Gage Calibration Wizard. Allow Enable Offset Nulling to remain checked and select Next.
d. The software automatically takes the first measurement, and the table displays the results. Select Calibrate. The Err % of the strain should now be a small value. Select Finish when done. When you click on the settings tab, you should now see a non-zero offset voltage that is applied to correct for the bridge imbalance at zero strain.

![Image of Measure and Calibrate window]

e. Select Ok to allow the DAQ Assistant to configure. Allow the VI to be placed in a loop when prompted. Modify the VI to display the strain as a numerical indicator, a waveform chart of the strain and the average and standard deviation of the measurement. Use a statistics block to generate the average and standard deviation. A typical block diagram is shown below.

To get the required items:

- Mathematics → Prob & Stat → S.D. & Variance
- Express → Sig Manip → From DDT (is automatically created when the S.D. and Variance is wired)
STOP: Show the system and VI to your TA. Demonstrate that the VI reads close to zero strain when the beam is unloaded.

4. Estimate Maximum Axial Strain for the 10lb Weight at the strain gage location
   a. Calculate the maximum bending moment $M_{\text{max}}$ and axial stress $\sigma_{\text{max}}$ in the beam at the strain gage location using the equations provided earlier in the lab manual. These maximum values will occur at the maximum applied load of 10 lb.

   b. Determine the material composition of your beam. Estimate maximum strain $\varepsilon_{\text{max-axial}}$, using Hooke’s Law and the given following values for the Young’s Modulus:

   $E_{\text{carbon steel}} \sim 28 \times 10^6$ psi (looks rusty), $E_{\text{stainless steel}} \sim 28 \times 10^6$ psi (looks thin and shiny), $E_{\text{alum}} \sim 10 \times 10^6$ psi (has a dull color and thicker), $E_{\text{titanium}} \sim 17 \times 10^6$ psi, and $E_{\text{magnesium}} \sim 7 \times 10^6$ psi (has a darker color)

5. Take Axial Strain Data
   a. Starting with no weights, record the axial strain value. Also, with 100 samples at 100 Hz, record the mean and standard deviation. Now add weights one pound at a time to the pan. Wait a few seconds between each reading so the beam and weight are in a steady state condition (the load is static). Continue to record the readings and to acquire the data as before.

   b. After 10 lbs have been added, remove the weights one pound at a time and record strain in a similar manner. Don't forget the zero-weight values at the end.
6. Repeat Above but using transverse strain gages
   a. Estimate the maximum transverse strain (similar to step 4) assuming $\nu = \frac{1}{3}$.

   **Note:** The length from the gages to the weight is different than before.

   b. Rewire the junction box for the transverse strain gauges (black banded connectors) and balance the bridge (calibrate) for $\varepsilon_{\text{trans}}$ measurement.

   c. Record the transverse strain data as in Step 5.

   **STOP**

   Plot the data in Excel (strain vs. applied load) for both axial and transverse strains. Does it look the way you expect (linear, zero strain at zero load, ratio of strains for axial to transverse are reasonable, sign?)

7. Measurement Uncertainty
   a. This experiment requires the estimation of the uncertainty of your results. You will need to estimate the uncertainty of all factors affecting your experiment. Recall the objective of this experiment. It was the estimation of the Young’s Modulus and Poisson Ratio by measuring the strains and calculating the stresses based on the applied load and beam geometry.
   b. Discuss with your lab partner(s) what factors will enter the uncertainty calculations (geometry, boundary conditions, loads, gauge factor, drift, accuracy, resolution).
   c. To estimate the uncertainty in the applied loads you can use the scale provided in the lab to weigh the weights used during the experiment. Record the resolution and accuracy of the scale together with the actual weights.
   d. Since the beam deforms and the boundary condition is not perfectly rigid, the applied load is not acting perpendicular to the beam. Use a protractor to estimate the angle at which the full load is applied. As measurement resolution record the actual resolution of the protractor or your ability to estimate the angle.
   e. Make sure you know the resolution of all the instruments you used to determine beam dimensions. Record that information at this point, if not recorded previously.

8. Clean up
   Carefully dismantle the equipment and return it to its original state for use by the next laboratory group
V. Required Data Analysis

1. Plot the calculated axial stress (y-axis) vs. the axial strain (given by the mean of the data acquisition measurements) for all of your weight trials both increasing and decreasing weight. Fit the data with a linear least-squares regression. Show the regression line and the individual data points on your plot. Determine the precision interval for the line at 95% probability.

\[ \sigma = a_0 + a_1 \varepsilon \pm t_{v,95\%} S_{\sigma \epsilon} \]

HINT: Use Excel, MATLAB or similar tool to generate this graph. Use a linear regression or the equations given in the statistics lecture to calculate the regression line. Refer to the example in the lecture notes. Recall that \( a_1 \) is the slope (= E-modul). Discuss the meaning of \( a_0 \) in your report.

2. The slope of the line (the \( a_1 \) coefficient from the fit) is your calculated modulus of elasticity. Evaluate the 95% precision interval of the measured modulus. This may be accomplished by finding the standard deviation of the slope of the fit, \( S_{a1} \), and the appropriate \( t_{95\%} \) factor.

\[ S_{a1} = S_{\sigma \epsilon} \left( \frac{N}{N \sum x_i^2 - (\sum x_i)^2} \right)^{1/2} \]

The uncertainty for \( a_1 \) is then given by \( a_1 \pm t_{v,95\%} * S_{a1} \). This is the precision interval for the E-modulus.

3. Using the modulus of elasticity from question 1 above, calculate the axial strain from the axial stress at the location of the transverse strain gages for each of the weight trials. Plot the transverse strain given by the mean of the data acquisition measurements vs. the axial strain calculated above. Fit the data with a linear least-squares regression and evaluate the 95% precision interval. The slope of the line is your Poisson's ratio. Use a similar procedure as in question 2 to evaluate the precision of your predicted value.

HINT: What is the stress at the location of the transverse strain gages? Why is it different from the stress at the axial strain gages? What is the uncertainty associated with the strain gage location?

For this question you calculate the axial strain from the axial stress and the estimated E-modulus from above. The transverse strain is measured via the strain gage.

\[ \nu_p = - \frac{\varepsilon_{\text{transverse}}}{\varepsilon_{\text{axial}}} \]

Again a plot can be generated based on axial and transverse strains. A linear regression yields the best fit and slope, which is the Poisson Ratio. Determine the confidence interval as before.

4. To find the total uncertainty in the modulus, you must determine sources of bias error. The calculated stress will have errors from uncertainties in the specimen width and height, the position of the strain gages, and the force applied by the mass, while the strain measurement will be affected by the accuracy of the gauge factor (including drift over the term of the experiment) and the accuracy and resolution of the data acquisition. There may also be
boundary conditions or other factors that will bias the result. Use the propagation of error technique to combine these individual uncertainties into a single bias uncertainty for modulus. Do this by writing the equation for modulus in terms of the parameters above and calculating the derivatives, paying close attention to units.

**HINT:** Since the beam deforms during the load application, the angle between the load direction and the beam axis deviates from 90 degrees. This can be included by considering a load angle \( \theta \).

Considering the load angle the equation for the E-modulus becomes

\[
E = \frac{\sigma_{\text{axial}}}{\varepsilon_{\text{axial}}} = \frac{6Fl}{bh^2} \left( \frac{G\nu}{E} \right) \cos \theta
\]

Recall from the lecture that the relative uncertainty \( e_i \) associated with each independent variable \( x_i \) is

\[
e_i = \left| \frac{\partial y}{\partial x_i} \right| u_{x_i}
\]

The total relative uncertainty \( e_y \) is

\[
e_y = \sqrt{e_1^2 + e_2^2 + \ldots + e_i^2}
\]

Using these equations determine the uncertainty for determining the modulus of elasticity \( E \). Calculate the uncertainty by following the examples given in the lecture notes.

5. Estimate the total uncertainty in the modulus of elasticity for your test specimen by combining your precision interval in modulus from step 1 with the bias error from step 4. Compare the result to published values for your specimen material.

**HINT:** Recall that the total uncertainty is obtained by combining bias error and precision error using the RSS method. Write an equation, which shows the 95% confidence interval for the E-modulus using the bias uncertainty from step 4 and the precision interval from step 2.

\[
E_{95\%\text{confidence}} = E_f \pm t_{(v, 95\%)} \frac{E_{\text{uncertainty}}}{v}
\]

6. The conclusions should state the modulus of elasticity and Poisson’s ratio found. Include the total error estimate for the modulus and the precision error for Poisson’s ratio. The comparison to reference values (cite the reference used) should also be presented along with discussion of possible differences between the measured and stated values. Is the theory for calculated stresses and strains (rather than measured ones) exactly correct for the experiment? Does the physical limitations of the experiment or boundary conditions affect the accuracy of the measured values? Recommend the most effective ways to improve the experiment, using both your error analysis and the discussion to justify your suggestions.

VI. References

1. Theory and Design for Mechanical Measurements, R.S. Figliola and D.E. Beasley.