The Pauli Exclusion Principle

Daniel Galehouse

University of Akron

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Outline

A short history

Molecular observations

Spinor Space

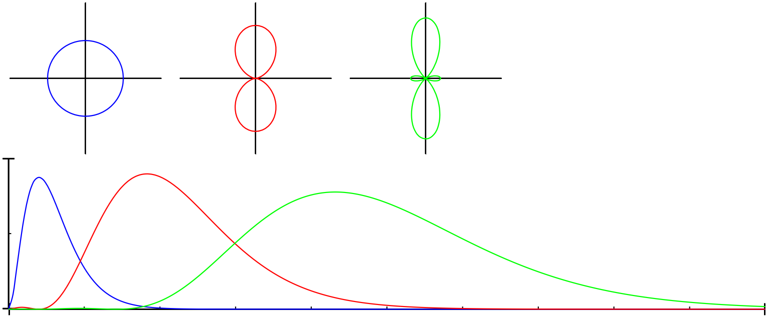
More than one electron

Ongoing questions

Mendeleev's periodic table

Reihen	Gruppe I. — R^2O	Gruppe II. — RO	Gruppe III. — R^2O^3	Gruppe IV. RH^4 RO^2	Gruppe V. RH^3 R^2O^5	Gruppe VI. RH^2 RO^3	Gruppe VII. RH R^2O^7	Gruppe VIII. — RO^4
1	H=1							
2	Li=7	Be=9,4	B=11	C=12	N=14	O=16	F=19	
3	Na=23	Mg=24	Al=27,3	Si=28	P=31	S=32	Cl=35,5	
4	K=39	Ca=40	—=44	Ti=48	V=51	Cr=52	Mn=55	Fe=56, Co=59, Ni=59, Cu=63
5	(Cu=63)	Zn=65	—=68	—=72	As=75	Se=78	Br=80	
6	Rb=85	Sr=87	?Yt=88	Zr=90	Nb=94	Mo=96	—=100	Ru=104, Rh=104, Pd=106, Ag=108
7	(Ag=108)	Cd=112	In=113	Sn=118	Sb=122	Te=125	J=127	
8	Cs=133	Ba=137	?Di=138	?Ce=140	—	—	—	— — — —
9	(—)	—	—	—	—	—	—	
10	—	—	?Er=178	?La=180	Ta=182	W=184	—	Os=195, Ir=197, Pt=198, Au=199
11	(Au=199)	Hg=200	Tl=204	Pb=207	Bi=208	—	—	
12	—	—	—	Th=231	—	U=240	—	— — — —

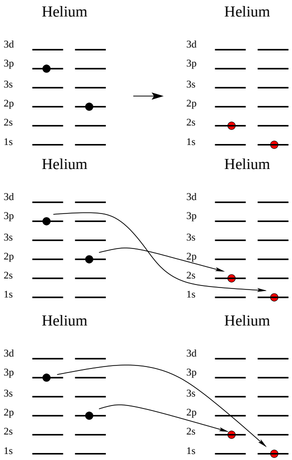
Quantum orbitals



Atomic states : (n, l, m, s)

Pauli: *Only one electron can be in each state.*

Identical particles



Symmetry types.

$$|m,n> \to |r,s>$$

$$|<m,n|r,s>|^2 = |<n,m|r,s>|^2$$

$$|m,n> = \pm|n,m> \begin{cases} \text{+ is Bose-Einstein} \\ \text{- is Fermi-Dirac} \end{cases}$$

If

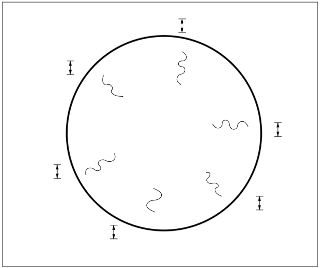
$$|m,n> = -|n,m> \text{ and } |p,s> = -|s,p>$$

Then

$$|n,m;p,s> = |m,n;s,p>$$

Bose-Einstein statistics

Blackbody radiation:



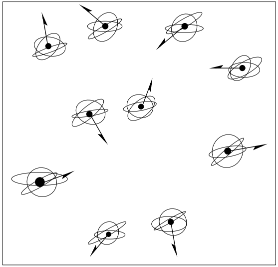
$$[a_\alpha, a_{\alpha'}] \equiv a_\alpha a_{\alpha'} - a_{\alpha'} a_\alpha = 0$$

$$[a_\alpha, a^\dagger_{\alpha'}] \equiv a^\dagger_\alpha a_{\alpha'} - a_{\alpha'} a^\dagger_\alpha = \delta_{\alpha\alpha'}$$

$$[a^\dagger_\alpha, a^\dagger_{\alpha'}] \equiv a^\dagger_\alpha a^\dagger_{\alpha'} - a^\dagger_{\alpha'} a^\dagger_\alpha = 0$$

Fermi-Dirac statistics

Spin gas:



$$\{b_\alpha, b_{\alpha'}\} \equiv b_\alpha b_{\alpha'} + b_{\alpha'} b_\alpha = 0$$
$$\{b_\alpha, b^\dagger_{\alpha'}\} \equiv b_\alpha b^\dagger_{\alpha'} + b^\dagger_{\alpha'} b_\alpha = \delta_{\alpha\alpha'}$$
$$\{b^\dagger_\alpha, b^\dagger_{\alpha'}\} \equiv b^\dagger_\alpha b^\dagger_{\alpha'} + b^\dagger_{\alpha'} b^\dagger_\alpha = 0$$

Dirac electron theory

Wave function:

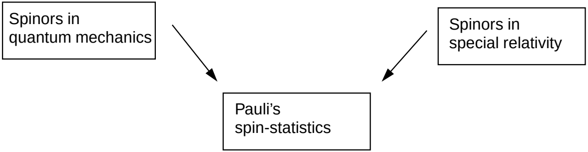
$$\Psi_A = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

Wave equation:

$$\left[\gamma^\mu \left(\frac{\partial}{\partial x^m} - \frac{eA_\mu}{c}\right) - mc\right] \psi = 0$$

Pauli's proof of exchange antisymmetry

Properties of two or more particle states are compared in quantum field theory.



Improper statistics are prohibited.

Spin and statistics history

1924	Stoner
1925	Dirac
1925-1946	Pauli
1925-1928	Jordan
1939	Belinfante
1939	Fierz
1940	deWet
1949-1965	Feynman
1951-1959	Schwinger
1957	Hall, Wightman, Streater
1957-1960	Jost
1958	Luders, Zumino
1958	Burgoyne

Neuenschwander's question

Question #7. The spin-statistics theorem

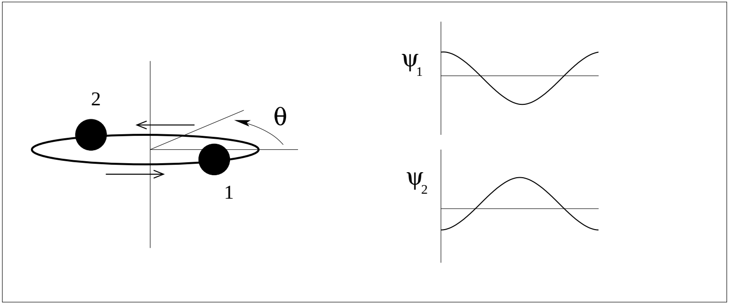
In the *Feynman Lectures on Physics*, Richard Feynman said[1]

"Why is it that particles with half-integral spin are Fermi particles whose amplitudes add with the minus sign, whereas particles with integral spin are Bose particles whose amplitudes add with the positive sign? We apologize for the fact that we cannot give you an elementary explanation. An explanation has been worked out by Pauli from complicated arguments of quantum field theory and relativity. He has shown that the two must necessarily go together, but we have not been able to find a way of reproducing his arguments on an elementary level... . This probably means that we do not have a complete understanding of the fundamental principle involved... ."

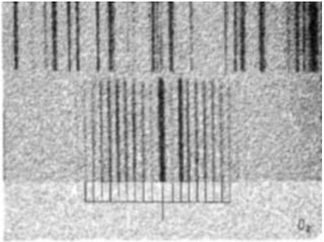
Has anyone made any progress toward an "elementary" argument for the spin-statistics theorem?

Dwight E. Neuenschwander
Physics Department
Southern Nazarene University
Bethany, Oklahoma 73008

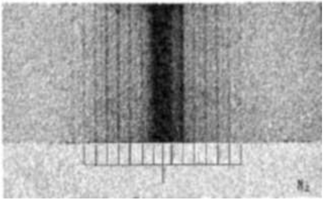
Molecular Spectroscopy



Rasetti



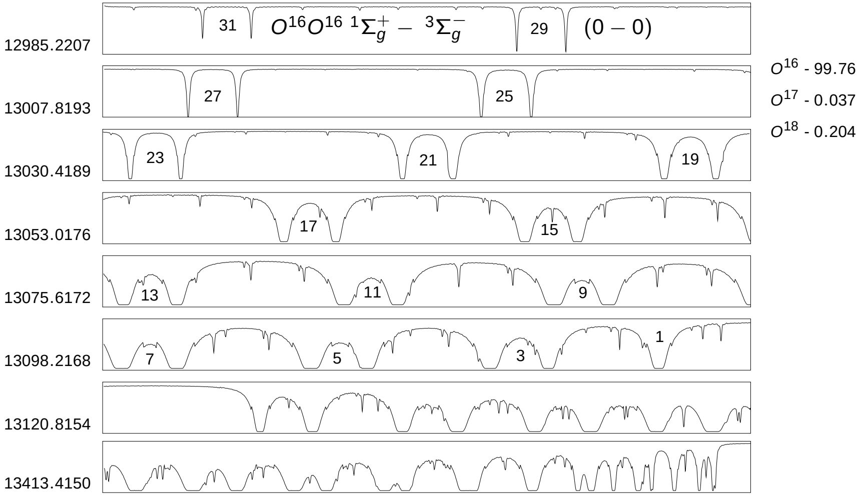
Alternate lines missing



Alternation of intensities

$I : I + 1 \to 1 : 2$

Modern observations



Evolution of spinor space

1968 Spin 1/2
1969 Gradient spinor
1971 Geometrical quantum theory
1974 Weyl theory
1980 Early five dimensional theory
1985 Quantum fields 5-D
1999 Source theory 5-D
2002 Early spinor space
2004 Spinor space field equation
2006 Neutrino theory
2008 Electron exclusion

Basic spinor space

Coordinates:

$$\xi = \xi^A \equiv (\xi_1, \xi_2, \xi_3, \xi_4) \equiv (\xi^{1r} + \xi^{1i}, \xi^{2r} + \xi^{2i}, \xi^{3r} + \xi^{3i}, \xi^{4r} + \xi^{4i})$$

Dirac spinor:

$$\Psi_A = \frac{\partial \Psi}{\partial \xi^A} = \begin{pmatrix} \partial\Psi/\partial\xi^1 \\ \partial\Psi/\partial\xi^2 \\ \partial\Psi/\partial\xi^3 \\ \partial\Psi/\partial\xi^4 \end{pmatrix}$$

Mapping:

$$\gamma^{\mu}_{AB} \longleftrightarrow x^{\mu}$$

Infintesimal relation:

$$dx^m = \zeta\gamma^m d\xi^\dagger + d\xi\gamma^{\dagger m}\zeta^\dagger$$

Conformal waves in spinor space

From 8-D Riemannian geometry: Ψ is a conformal wave

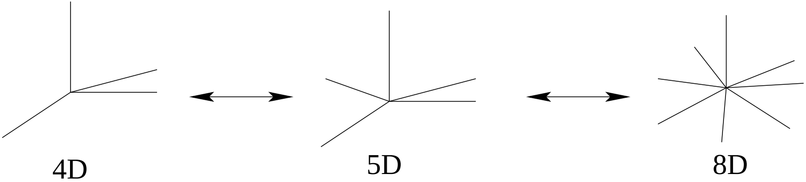
$$0 = \boxtimes \Psi \equiv \frac{\partial}{\partial \xi_A} \frac{\partial}{\partial \xi^A} \equiv$$

$$\left[\left(\frac{\partial}{\partial \xi_1} \right)^2 - \left(\frac{\partial}{\partial \xi_2} \right)^2 + \left(\frac{\partial}{\partial \xi_3} \right)^2 - \left(\frac{\partial}{\partial \xi_4} \right)^2 - \left(\frac{\partial}{\partial \xi_5} \right)^2 + \left(\frac{\partial}{\partial \xi_6} \right)^2 - \left(\frac{\partial}{\partial \xi_7} \right)^2 + \left(\frac{\partial}{\partial \xi_8} \right)^2 \right] \Psi$$

gives according to the chain rule

$$\zeta \gamma^m \frac{\partial \Psi}{\partial x^m} = 0.$$

Geometrical interactions



$$\frac{1}{2}\{\gamma^m, \gamma^n\} \equiv \gamma^{mn} \equiv \begin{pmatrix} g_{\mu\nu} - \mathcal{A}_\mu \mathcal{A}_\nu & -\mathcal{A}_\mu \\ -\mathcal{A}_\nu & -1 \end{pmatrix}$$

Local electron

Match plane waves in different dimensionalities

$$\psi = e^{i(\vec{k}\vec{x} - \omega t)}$$

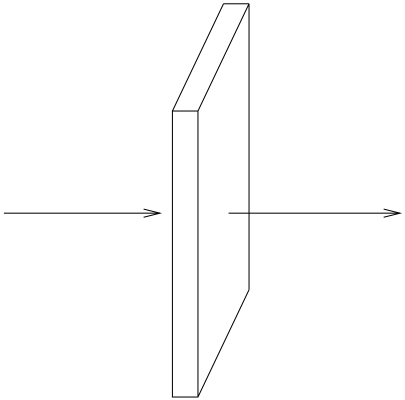
$$\updownarrow$$

$$\Psi = e^{i(\vec{k}\vec{x} - \omega t - m\tau)}$$

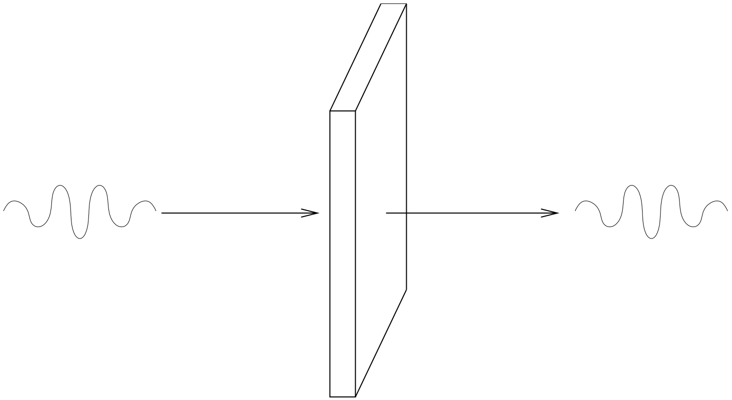
$$\updownarrow$$

$$\Psi = e^{i\kappa_A \xi^A}$$

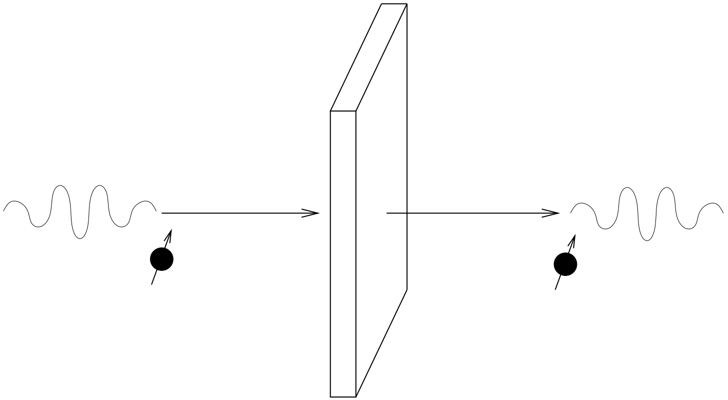
Spinor space I: simple particle



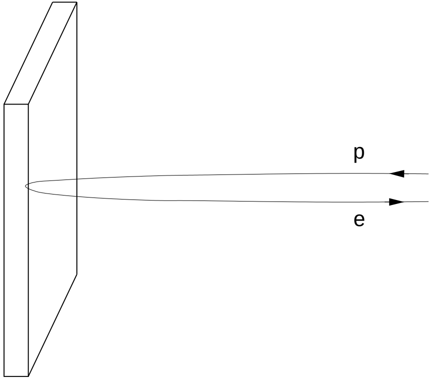
Spinor space II: wave particle



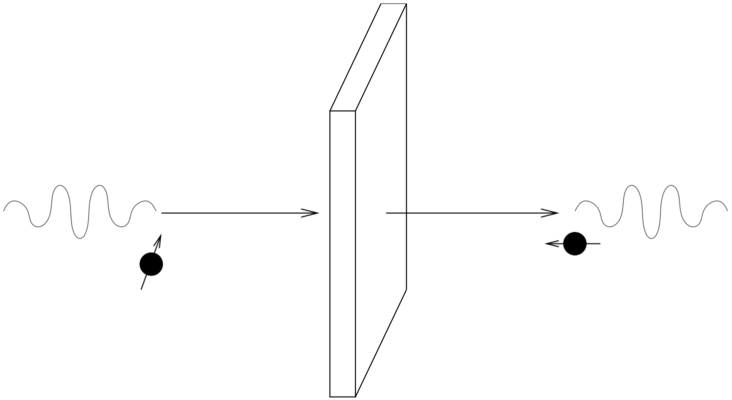
Spinor space III: wave particle with spin



Spinor space IV: pair production



Spin space V: weak interaction?



Escape!

Theoretical and experimental aspects of the spin-statistics connection and related symmetries
Stazione Marittima Conference Center
Trieste, Italy
21-25 October 2008
International Advisory Committee
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- Sergio Bertolucci (LNF-INFN, Italy)
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phone: +39 040 558 3367
fax: +39 040 558 3350
e-mail: spinstat2008@ts.infn.it
The workshop will focus on the spin-statistics connection and on related symmetries, both from the theoretical and from the experimental point of view. The workshop will bring together experimentalists, theorists, and philosophers to survey work done during the past years that challenges the traditional views of these issues. The workshop will also explore connections with rapidly developing fields such as other fundamental symmetries, supersymmetry and quantum gravity.
Background pictures: Wolfgang Pauli in 1924, approximately at the time he thought about a primitive form of the exclusion principle and on its role in the alkali atom spectra (Photo 11 from the Pauli Archive at CERN, © CERN Copyright, used with permission). The colored lines are a sodium gas emission spectrum.)
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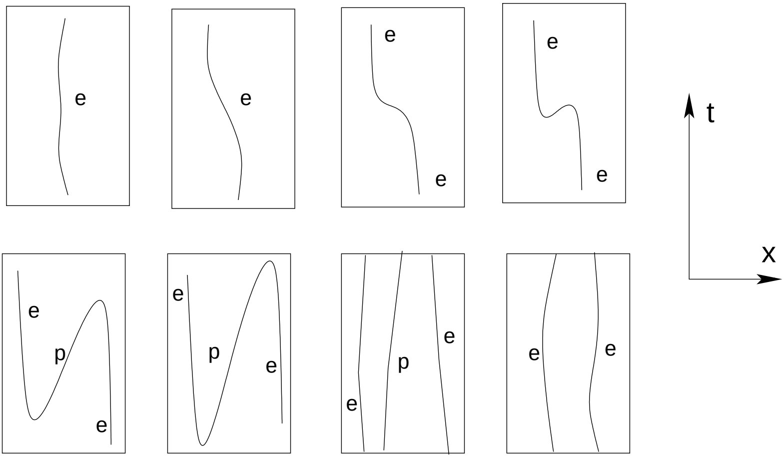
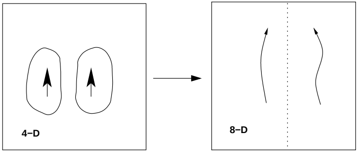
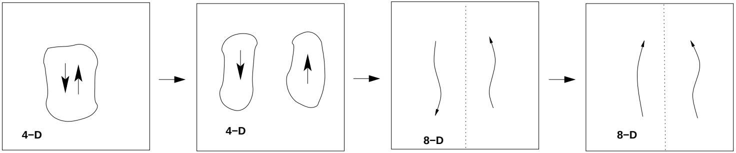
An identified pair



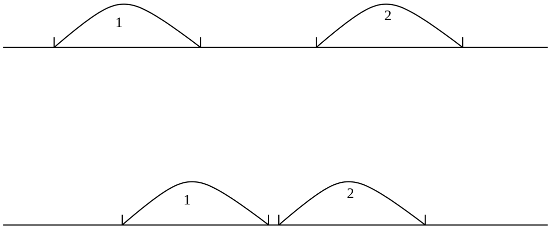
Parallel electrons



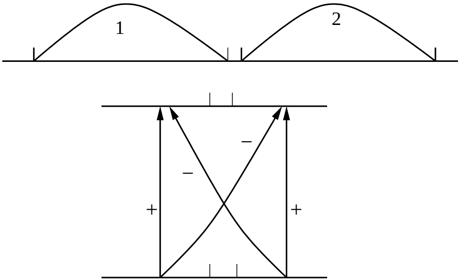
Anti-parallel electrons



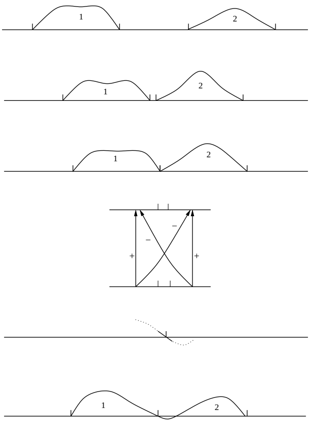
Spinor wave propagation



Boundary development



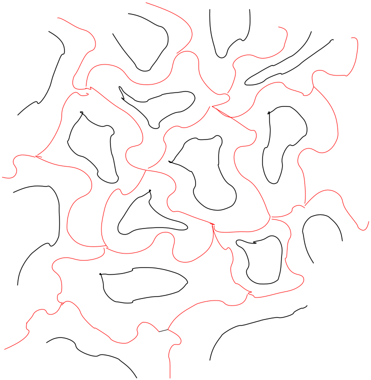
Wave function antisymmetrization



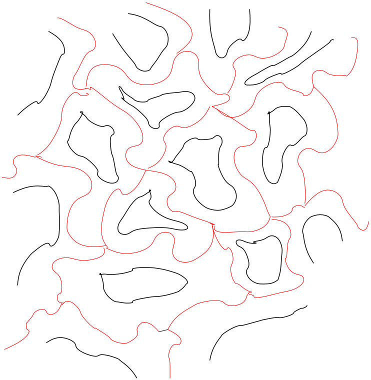
$$\Psi_A = \frac{\partial \Psi}{\partial \xi^A}$$

$$\Box \Psi = 0$$

Multiple electrons in spinor space

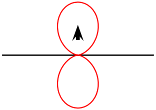


Multiple electrons in spinor space



Orbitals in spinor space

What does a typical orbital look like in spinor space?



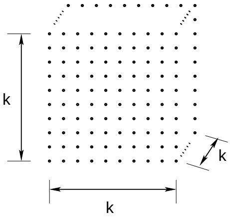
→ ?

Space-time orbital Spinor space orbital

What do the hydrogen atom solutions look like?

Calculational complexity

The wavefunction of a single electron may be represented on a grid of points.

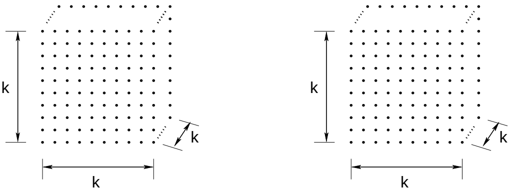


In spacetime, choose
$N = k \cdot k \cdot k = k^3$
In spinor space, perhaps
$N = k \cdot k \cdot k = k^3$

A collection of parameters are chosen for each electron.

Calculational complexity in Fock space

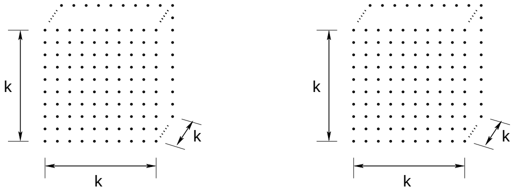
Antisymmetry is explicit.



$\psi(r_1, r_2) = \psi_1(r_1)\psi_2(r_2) - \psi_2(r_1)\psi_1(r_2)$, N= $k^3 \cdot k^3$.
For $k > 2$,
$\psi(r_1, r_2, \cdots r_k) = \Sigma_A sgn(A) \cdot A\binom{r_1, r_2, \cdots r_k}{w_1, w_2, \cdots w_k}\psi(w_1, w_2, \cdots w_k)$,
$N = (k^3)^n$.

Separate electrons in spinor space

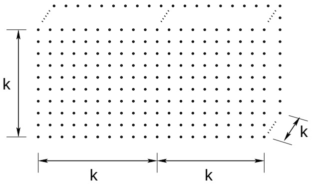
In spinor space, no explicit antisymmetrization is needed.



The effect appears when the electrons meet.

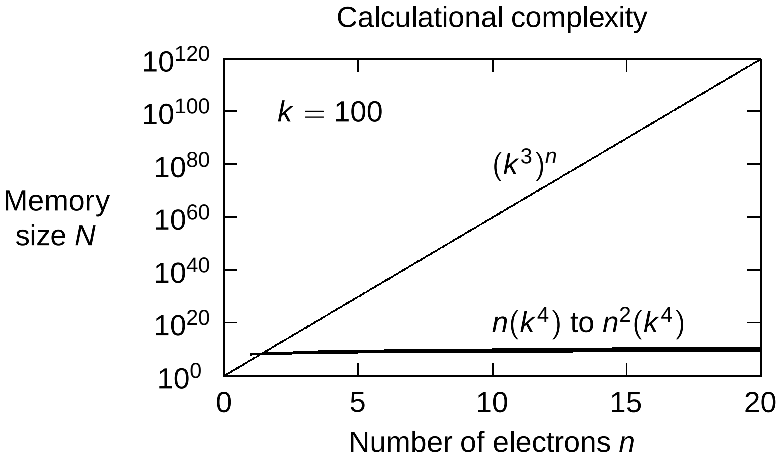
Calculational difficulty in spinor space

Antisymmetrization forms at the boundary.

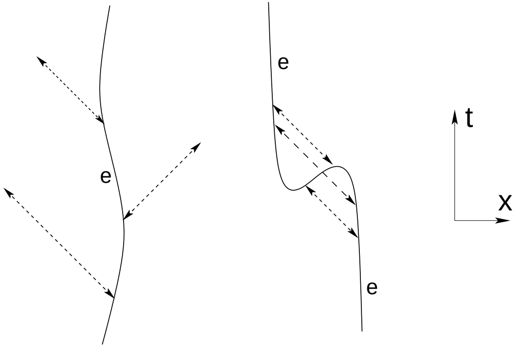


The interface need not be simple.

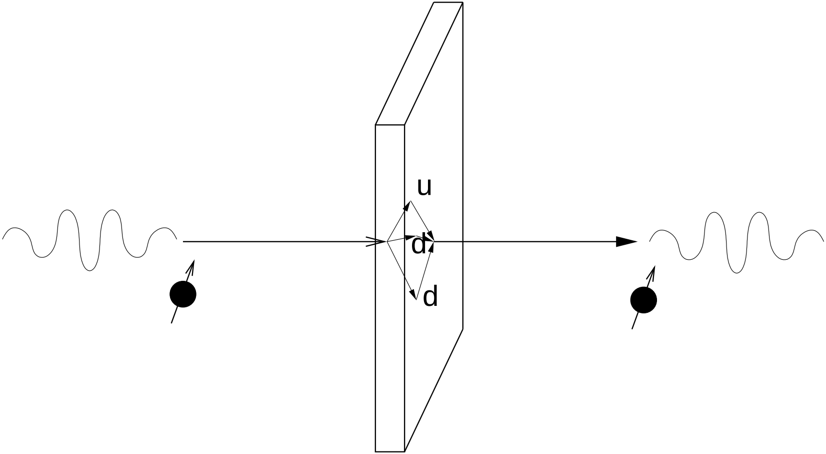
Comparison


Calculational complexity
10^{120}
10^{100}
10^{80}
10^{60}
10^{40}
10^{20}
10^{0}
Memory size N
$k = 100$
$(k^3)^n$
$n(k^4)$ to $n^2(k^4)$
0
5
10
15
20
Number of electrons n

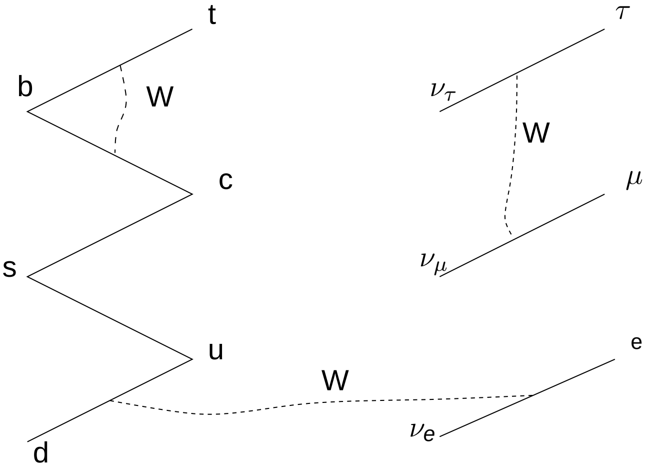
Alpha



Quarks



Extra exclusion



Essential quantum terms.

$$H = \frac{p^2}{2m} + V(x)$$

$$H \to -i\hbar\frac{\partial}{\partial t}, p \to i\hbar\frac{\partial}{\partial x}$$

$$-i\hbar\frac{\partial\psi}{\partial t} = \frac{1}{2m}\left(i\hbar\frac{\partial}{\partial x}\right)^2\psi + V(x)\psi, \quad \psi = \exp(\frac{iS}{\hbar})$$

$$\frac{\partial S}{\partial t} = \frac{1}{2m}\left(\frac{\partial S}{\partial x}\right)^2 - i\beta\hbar\frac{\partial^2 S}{\partial x^2} + V(x)$$