Pauli's Exclusion Principle in Spinor Coordinate Space

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Theoretical and Experimental aspects of the Spin Statistics connections and related symmetries, 2008

Outline

The problem of derivatives.

- Matrix mechanics

$$\mathbf{pq} - \mathbf{qp} = -i\hbar$$

- Wave mechanics

$$\frac{\partial}{\partial q} q - q \frac{\partial}{\partial q} = 1$$

- General relativity

$$D_j \Phi^i = \Phi^i_{;j} = \frac{\partial \Phi^i}{\partial x^j} + \Gamma^i_{jk} \Phi^k$$

Conformal waves

Wave equations from the Riemann tensor.

- Let the conformal factor be Ψ^p with $p = 4/(n-2)$.
- Ψ obeys a linear wave equation in n dimensions.

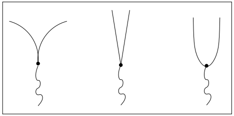
$$\frac{\partial^2 \psi}{\partial x^a \partial x_a} = R = 0$$

Quantum field equation.

In five dimensions.

$$\frac{1}{\sqrt{-\dot{g}}}(i\hbar\frac{\partial}{\partial x^\mu} - eA_\mu)\sqrt{-\dot{g}}g^{\mu\nu}(i\hbar\frac{\partial}{\partial x^\nu} - eA_\nu)\psi =$$

$$[m^2 + \frac{3}{16}(\dot{R} - \frac{e^2}{4m^2}F_{\alpha\beta}F^{\alpha\beta})]\psi$$



Interaction mechanism

- Conformal mediation

$$R_{ij}(\omega\gamma^{mn}) = 0 \quad \rightarrow \quad R_{ij}(\gamma^{mn}) = T_{ij}$$

- Gravitational source equations

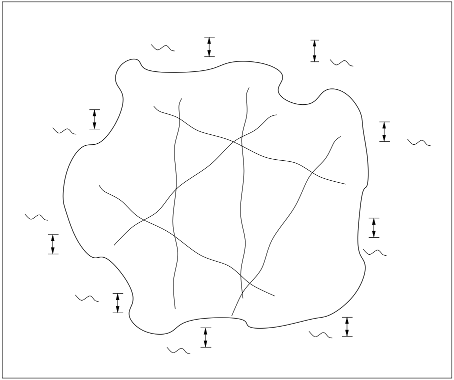
$$R^{\alpha\beta} = 8\pi\kappa\left[F^{\alpha}{}_{\mu}F^{\mu\beta} + m|\psi|^2\frac{e^2}{m^2}A^{\alpha}A^{\beta} + m|\psi|^2\frac{1-(e^2/m^2)A^2}{2-(e^2/m^2)A^2}g^{\alpha\beta}\right]$$

- Electromagnetic source equation

$$F^{\beta\mu}|_{\mu} = 4\pi e|\psi|^2 A^{\beta}$$

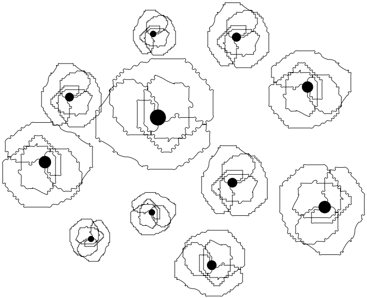
Second quantization of photons and gravitons

$$A_\mu = A_\mu(\text{ret.}) + A_\mu(\text{adv.})$$



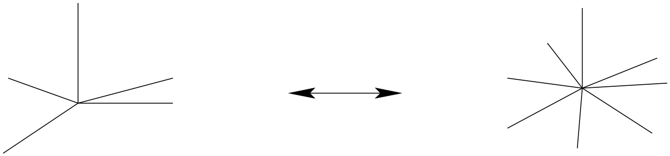
Second quantization of electrons

Specific heat of a monatomic gas, spectroscopy



$$\{b_\alpha, b_{\alpha'}\} = 0$$
$$\{b_\alpha, b^\dagger_{\alpha'}\} = \delta_{\alpha\alpha'}$$
$$\{b^\dagger_\alpha, b^\dagger_{\alpha'}\} = 0$$

Local definition of spinor coordinates.



$$\xi^{A} = \xi^{A}_{r} + i\xi^{A}_{i}, \qquad \xi^{\bar{A}} = \xi^{\bar{A}}_{r} - i\xi^{A}_{i}, \qquad A = 1 \cdots 4$$

$$\epsilon_{A\bar{B}} = \epsilon^{A\bar{B}} = \mathrm{diag}(1, 1, -1, -1)$$

$$dx^{m} = \zeta^{A}\gamma^{m}{}_{A}{}^{B} d\xi^{\bar{C}}\epsilon_{\bar{C}B} + d\xi^{A}\bar{\gamma}^{mB}{}_{A}\zeta^{\bar{C}}\epsilon_{\bar{C}B} \equiv \zeta\gamma^{m}d\xi^{\dagger} + d\xi\gamma^{\dagger m}\zeta^{\dagger}$$

Conformal Waves in spinor space

Using, for the Dirac wave function,

$$\Psi_B = \frac{\partial \Psi}{\partial \xi^B}$$

if Ψ is a function in extended space-time, the conformal wave

$$0 = \boxtimes \Psi \equiv \epsilon^{\bar{A}B} \frac{\partial}{\partial \xi^{\bar{A}}} \frac{\partial}{\partial \xi^B} \Psi \equiv \epsilon^{\bar{A}B} \frac{\partial \Psi_B}{\partial \xi^{\bar{A}}}$$

gives according to the chain rule, the Dirac equation

$$\zeta^D \left[\gamma^m{}_D{}^B \frac{\partial \Psi_B}{\partial x^m} \right] = 0$$

Local Dirac electron

A plane wave in five space

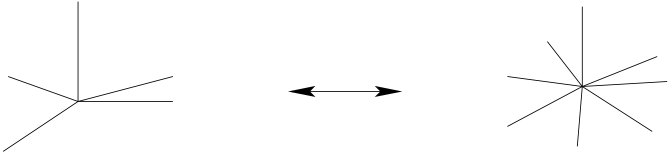
$$\Psi = e^{i(\vec{k}\vec{x} - \omega t - m\tau)} \equiv e^{ik_m x^m}, \qquad k_m = (\vec{k}, \omega, m)$$

becomes after differentiation in spinor space

$$\Psi_A \equiv \frac{\partial \Psi}{\partial \xi^A} = \Psi i k_m \frac{\partial x^m}{\partial \xi^A} \quad \Rightarrow$$

$$i\Psi k_m \gamma^{\dagger m} \zeta^\dagger = i\Psi \begin{pmatrix} k_0 & 0 & im - k_3 & -k_1 + ik_2 \\ 0 & k_0 & -k_1 - ik_2 & im + k_3 \\ im + k_3 & k_1 - ik_2 & -k_0 & 0 \\ k_1 + ik_2 & im - k_3 & 0 & -k_0 \end{pmatrix} \zeta^\dagger$$

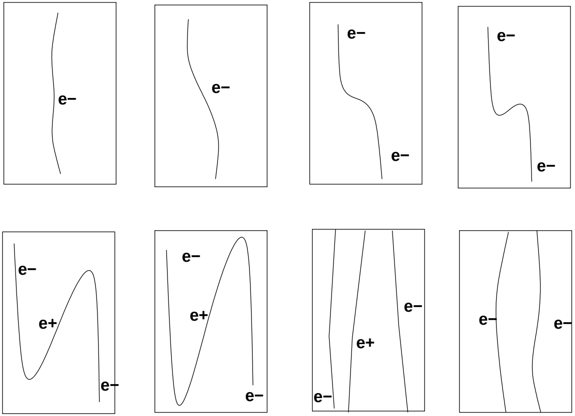
Transformation theory of interaction



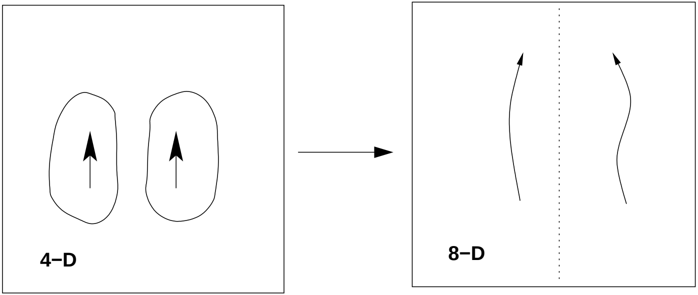
$$\frac{1}{2}\{\gamma^m, \gamma^n\} \equiv \frac{1}{2}(\gamma^m\gamma^n + \gamma^n\gamma^m) =$$

$$\gamma^{mn} \equiv \begin{pmatrix} g_{\mu\nu} - \mathcal{A}_\mu \mathcal{A}_\nu & -\mathcal{A}_\mu \\ -\mathcal{A}_\nu & -1 \end{pmatrix}$$

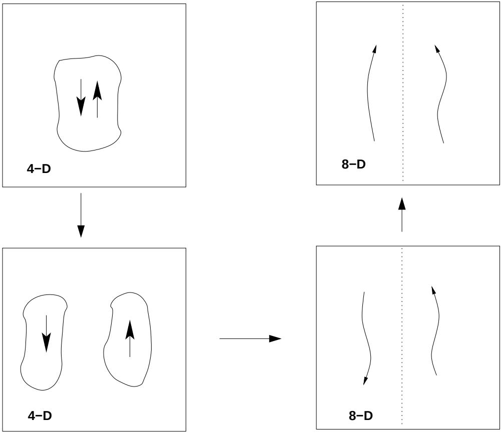
An identified pair



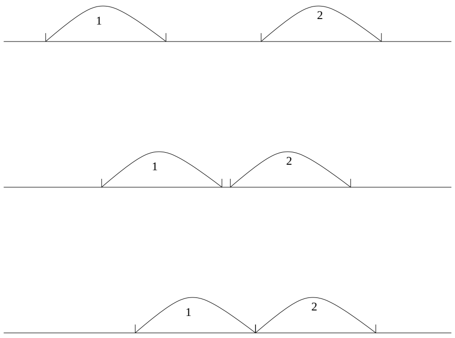
Parallel electrons



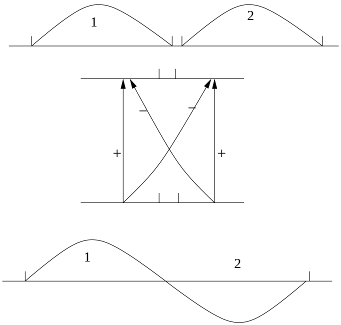
Anti-parallel electrons



Spinor wave propagation



Boundary development



Standard boundary conditons:

$$\psi'(1) = a[\psi(1) - \psi(2)]$$

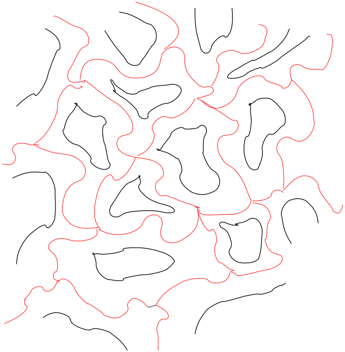
$$\psi'(2) = a[\psi(2) - \psi(1)]$$

Spinor coordinate boundary condition:

$$\Psi_A = \frac{\partial \Psi}{\partial \xi^A}$$

$$\boxtimes \Psi = 0$$

Multiple electrons in spinor space



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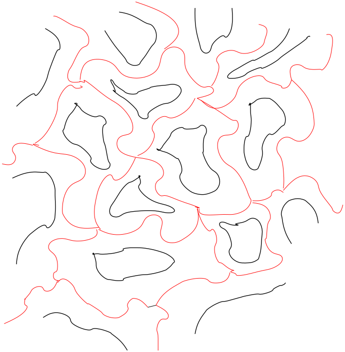

Ongoing considerations

- Questions and problems
  - Calculational advantages
  - Relativistic formalism, Feynman exchange
  - Interparticle interaction/self-interaction
  - Operators
  - Other Fermions
  - Dirac-Thirring paradox, rotation in G.R.
  - Newton's bucket
  - Aharonov-Casher

Geometry of the Pauli Equivalence Principle

- The geometrical description of fundamental physics.
- The natural relevance of spinor coordiantes for electrons.
- The elementary description of the Pauli equivalence principle as a property of differential equations.

References

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