

Quantum Geometrical Theory, Interaction and Spin

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1 Introduction

A geometrical theory is presented that combines quantum mechanics with electrodynamics and gravity in a self-consistent and fully covariant form. A number of conventional ideas must be revised and some new ones added to allow for quantum effects in gravitational fields. Careful interpretation permits a particle by particle description that is mathematically determined. This allows a deterministic geometry to include a basis for quantum statistics in radiating systems. Motion and density are natural parts of the geometry. Each particle has a geometrical description that includes a wave function, vector potential and metric. These quantities combine to interact in five dimensions giving the know laws of particle behavior in space-time. An explicit dimensional reduction scheme is not required. The system satisfies full equivalence, and all forces can be interchanged by geometrical transformations. Further advances are expected. Spinors, which are know to be intrinsically geometrical, seem compatible and should bring in the weak interactions. Extension to the standard model may be possible. It is an ongoing study to find a fundamental geometry that includes all of physics.

2 Search for a Fundamental Geometry

The initial goal is to discover an elementary but fully geometrical quantum mechanics that includes all known quantum properties. This is to be a simple incorporation of essential characteristics without a commitment to any specific interpretation. Such an elementary understanding should lead to reconciliation with relativity and gravity. Given the divergent metaphysical issues within the relevant subfields, correctness must be resolved by recourse to experimental results without any predisposed phenomenology.

It is easy to understand that electrodynamics has to be included. While there are textbook examples of isolated quantum wave functions, practical quantum effects are almost always observed by electromagnetic interactions. But the inclusion of radiation requires further sophistication. Because radiation is inherently relativistic, special relativity must be incorporated, a fact that is ignored in most non-relativistic quantum theories. The issues of quantum-electrodynamics become

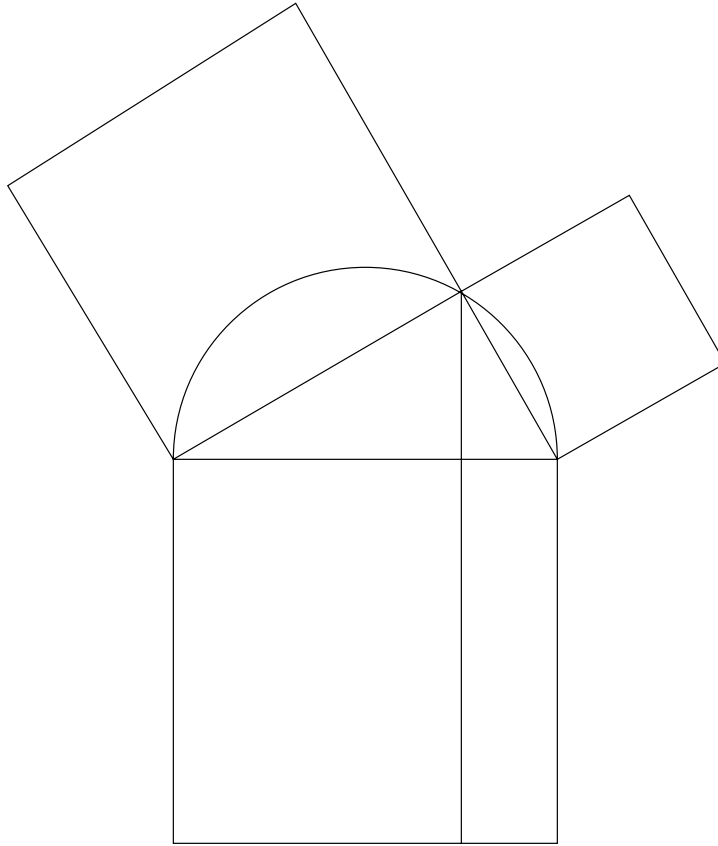


Figure 1: The fundamental concepts of geometry come from the material properties of physical objects. Practical experience with measuring instruments leads to the abstract notions of Euclid.

relevant. Because of relativistic constraints, potentials that depend on space-like separated points must be relinquished in favor of covariant forces that act along null lines. Simple instructional examples of any Lorentz covariant quantum interactions are hard to find. The phenomenology and the concomitant mathematical representation are complicated.

Any but the simplest interactions, when presented in a relativistic setting, require a curvilinear description. In general relativity, the clock paradox is often used to motivate the higher formalism. Not so obvious is that this difficulty occurs with other interactions. A Lorentz frame describes only the simplest motion yet, geometrical equivalence must be maintained. While an explicit physical “clock” paradox may not be available, Lorentz coordinate transformations are inadequate. Any suitable construction requires curvilinear coordinates, and a Riemannian formalism follows. Realizing this is essential to allow for the description of physical interactions in a quantum-relativistic setting¹.

¹See D. Galehouse, A Fundamental Geometry of Quantum Physics, *Balkan Journal of Geometry and its Applications* **5**, no.1, (2000),93–106 and the references contained therein.

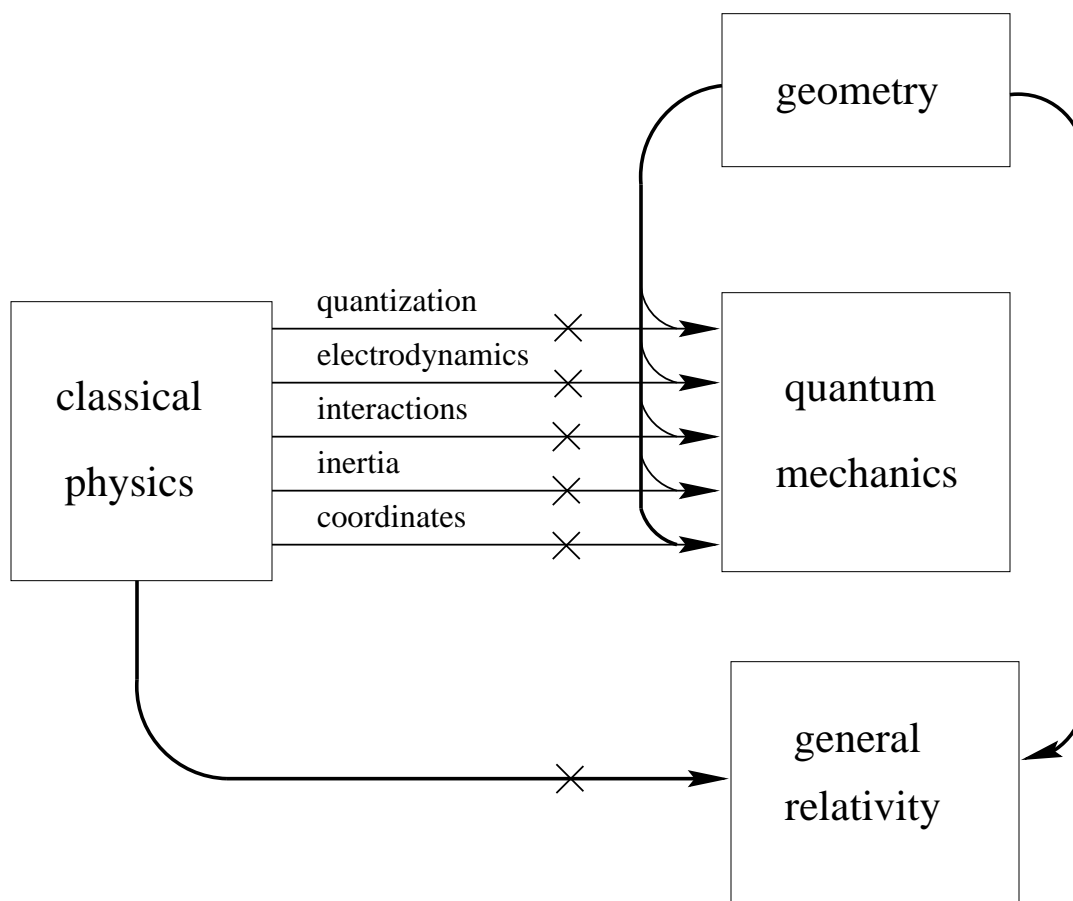


Figure 2: A geometrical theory displaces classical physics as a fundamental theory and, in particular, as a progenitor of quantum theory

3 Epistemology and Geometrization

Attempts to combine quantum systems with gravitation have shown unresolved difficulties. The root causes are seen here to originate in the metaphysical assumptions of both subfields. New ideas are necessary and some old ones may be untenable. By the usual convention, quantum mechanics is derived from classical mechanics by quantization. A classical Hamiltonian becomes a quantum wave equation by substitution of derivatives for algebraic quantities. The process brings the electrodynamic interaction across as well. The geometrical sense of coordinates is unchanged and the partial differential equations of quantum theory come into existence.

In contrast to this, general relativity has a fundamental geometrical basis outside of classical mechanics. Herein is the conflict; some physics has a geometrical

component while some has not. Resolution of these differences is essential².

As a progenitor of quantum mechanics, geometry may displace classical physics. Studies show that some of the essential quantum theory is buried in curvilinear geometrical constructions. Apparently, formal quantization is not always needed, and certain essential parts of quantum theory are found in geometries unconverted by quantization. A major change in the epistemological structure of physics is proposed. Classical physics is to be a simple phenomenology and should not be used to generate quantum theory. The quantum should come from geometry alone.

This is a realistic possibility because there are no experiments that support the classical over the quantum mechanics. As there are no observed point particles, there can be no fundamental basis for the older theory. Many present difficulties come about because the acceptance of active quantization allows inconsistencies. It is a root difficulty. It is overly optimistic to start with a fundamentally incorrect classical theory and expect to arrive, by any rigorous mathematical process, at a correct quantum theory. Avoiding this dependency allows useful unified field theories to emerge naturally.

4 Quantization

A review of the procedure for the conversion of a classical theory to a quantum theory makes the limitations more apparent. The common strategy is to start with an algebraic relation for the Hamiltonian such as,

$$H = \frac{p^2}{2m} + V(x) \quad (1)$$

and convert it to

$$-i\hbar \frac{\partial \psi}{\partial t} = \frac{1}{2m} \left(i\hbar \frac{\partial}{\partial x} \right)^2 \psi + V(x)\psi, \quad (2)$$

by the substitution rule

$$H \rightarrow -i\hbar \frac{\partial}{\partial t} \quad (3)$$

$$p \rightarrow i\hbar \frac{\partial}{\partial x} \quad (4)$$

The resulting equation, under restricted circumstances, is in agreement with experiment. The mathematical process has been studied for some time, but never formally validated³.

²See the archived preprint, D. C. Galehouse, "Quantization failure in Unified Field Theories", xxx.lanl.gov quant-th/9412012

³See P. Havas, *Bull. Am. Phys. Soc.* **1**, (1956) 337–338, and F. J. Kennedy and E. J. Kerner, *American Journal of Physics*, **33** (1965) 463–466, **34** (1966) 271.

To be explicit about the difficulties, consider the quantum equation,(2), substitutionally transformed by

$$\psi = e^{iS/\hbar} \quad (5)$$

with $S = S(x, t)$ a complex function of space-time. The result is

$$\frac{\partial S}{\partial t} = \frac{1}{2m} \left(\frac{\partial S}{\partial x} \right)^2 - i\beta\hbar \frac{\partial^2 S}{\partial x^2} \quad (6)$$

where $\beta\hbar$ has been written in for β .

When $\beta = 0$ the standard classical equation is obtained and when $\beta = 1$ it is the corresponding quantum version. The multiplying term of β , $-i\hbar \frac{\partial^2 S}{\partial x^2}$ is here called an essential quantum term. It is of first order according to the power of β . The enigma occurs when β has an intermediate value, for then predictions of the intermediate theory agree with experiment better than classical physics. If this is true, how can we know that the classical formulae are unique? It surely a condition for fundamentality. In fact, only the deficiency of classical measurement standards allows the effects of the essential quantum terms to go unnoticed. The specific theory that we call classical physics is not objectively valid but is accepted only because of its history and simplicity. Complicating this discussion is that the classical system allows descriptions of physical effects by point objects, a result that cannot be obtained for $\beta \neq 0$. The existence of such point objects has never been established experimentally and the deficiency is artificial. The weakness in the quantization paradigm is that there is no fundamental objective starting point. Apparently, the result of quantization can be correct in some cases. However, if there are errors in both the method and the starting point, misleading results may occur.

In a curvilinear system the difficulties can be made plain. The usual method requires that specific differential operators be substituted for energy and momentum. But, there are already derivatives that lead to corrections indistinguishable from the essential quantum terms. It is not possible to assign the quantum effects unambiguously. Specifically, the derivatives of the metric tensor are implicit in the definition of covariance. They persist in the connections

$$\Gamma_{\beta\gamma}^{\alpha} = \frac{1}{2} g^{\alpha\delta} \left(\frac{\partial g_{\delta\beta}}{\partial x^{\gamma}} + \frac{\partial g_{\delta\gamma}}{\partial x^{\beta}} - \frac{\partial g_{\beta\gamma}}{\partial x^{\delta}} \right) \quad (7)$$

which are part of any curvilinear structure. These terms are essential for a gravitational field. A substitutionally introduced derivative may operate on a scalar, but the covariant application to tensors or spinors is problematic. No resolution has become available. And, associated problems with quantum normal ordering persist. An accepted classical theory must contain the momentum p and energy E in such a way as to transform covariantly. Because the Christoffel symbols are required, a purely algebraic classical precursor does not exist, and the theory cannot be quantized. Apparently, any ‘‘correct’’ classical gravitational theory cannot be converted

to a correct quantum theory. The introduction of additional derivatives destroys the covariance if even the method can be defined unambiguously.

Moreover, the characteristic gravitational field equations are already second order. A quantization would increase the order, by at least one and probably two. The resulting equations are third or fourth order, even neglecting complications generated by the non-locality of possible expansions of derivatives under the square root. This is an epistemological disaster because this conventional method to establish a quantum-gravitational theory cannot be made intellectually sound. The proposed recourse is to use covariant quantities to represent the wave function and generate field equations by the selection of a geometrical system.

5 Quantum Conformal Coupling

Of additional significance is the natural relationship of quantum particle density to the conformal factor of the space-time metric. Observations of the quantum density are confounded by the geometrical density. Again, an illustrative example will suffice. Each observation of a probability might involve counting particles as they are detected on a test screen. The experimental arrangement is controlled by material objects which serve as measuring rods. In this sense the apparatus is fixed and unchanging. A region on the screen is scribed to delineate an area in which particles will be counted as they are detected. The marked area is measured to have dimensions δx and δy and the particle movement by δz and δt , all according to local metric $\dot{g}_{\mu\nu}$. A count of particles, as given by quantum theory, is

$$N = \psi\psi^* \frac{\delta x \delta y \delta z}{\delta t}$$

Consider now the observation of this effect by an observer using a metric having a different conformal parameter and consequently a different scale, $\dot{g}'_{\mu\nu} = \lambda \dot{g}_{\mu\nu}$. The new numerical marks are

$$(\delta x', \delta y', \delta z', \delta t') = \sqrt{\lambda} \cdot (\delta x, \delta y, \delta z, \delta t). \quad (8)$$

The count becomes

$$N = \frac{\psi\psi^*}{\lambda} \frac{\delta x' \delta y' \delta z'}{\delta t'} \quad (9)$$

and because N is an invariant, ψ must not be taken unchanged. The probability density and wave function must be compensated. Presumably, $\sqrt{\lambda}\psi' = \psi$, and the conformal structure of the geometry is linked to the quantum mechanical density. Many of the essential effects of quantum mechanics are manifestations of conformal structure. A quantitative description of particle localization is identified with the relative conformal density between the particle space and the observer.

A conformally invariant theory prevents this coupling and suppresses the intrinsic quantum characteristics. Such a theory cannot be quantized because the

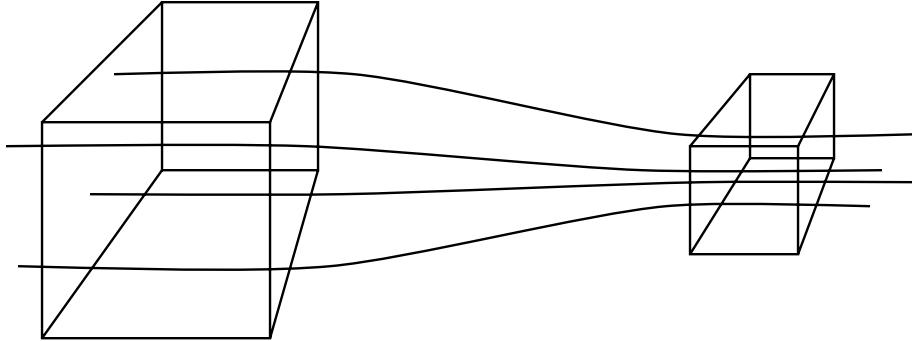


Figure 3: Local variations in the conformal parameter of a particle space-time are equivalent to probability density waves.

geometrical density is isolated from the quantum density. A geometrical interpretation requires a coupling to the conformal factor.

The derivatives within the Christoffel symbols operate on the metric. Dependence on the conformal parameter is implicit and derivatives of the wave function are implied. The ambiguity between these derivatives and any that might be introduced by active quantization persists and prohibits a successful result.

6 Wave-particle Duality

A deterministic and geometrical quantum theory impels the resolution of wave-particle duality. Conventional quantum wisdom says that real particles behave sometimes like particles and sometimes like waves. Double-talk notwithstanding, the description must be made more precise and the language adjusted to the reality of experiments.

A precise notion of a classical point particle is required. It is taken here to be an object that is described by a one dimensional trajectory (geodesic) in space-time. There may be additional parameters, such as mass or charge, that are part of the dynamics. Such is the common notion.

A more sophisticated concept is required for the quantum case. A one-dimensional trajectory is never sufficient. Augmenting the confusion is the accepted terminology that the distinguishing characteristic of a quantum point object is simply that it is non-composite. Electrons are ascribed to be quantum point-like while protons or pions are not, this, even though none can be condensed to a point. In reality, the essential idea of a quantum *point* particle is simply the cardinality of particle number. Electrons, by experiment, come in sets that have a total number associated. The localization is irrelevant. The enumeration may be defined by weight, inertia, charge, or other interaction. This notion of number is adequate for quantum me-

chanics and allows for normalization of the wave function. Experiments show that, classical physics notwithstanding, available particles are countable but never point-localized. There is not now, nor has there ever been any experimental evidence for a finite amount of mass, or charge, actually concentrated at a mathematical point.

Quantization subtly integrates the classical idea of a point particle into the quantum interpretation. The quantum structure may be corrupted. The paradox of wave particle duality then comes into play. To illustrate, consider a diffraction experiment designed to elucidate the physical measurement of the probability density.

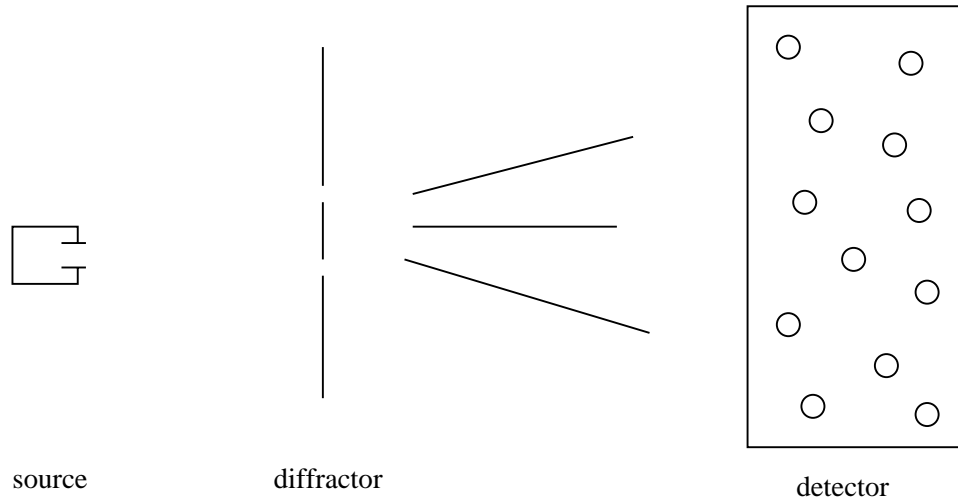


Figure 4: Diffracted electrons are captured in a detector. An explicit model for the detector demonstrates the evolution of the wave function from an extended diverging wave to a compact bound state.

Particles travel through a diffraction screen to be counted by a detector. The detector consists of a fixed uniform distribution of charged centers, (perhaps protons), to which the electrons are attracted. An individual electron will interact with these charged objects, emit radiation, lose energy and ultimately be captured. Under ideal conditions, it will bind to one of the charged centers. The large initial diffraction pattern will condense into an atomic wave function. Because of the multiple outcomes, this process generates statistics. The particular final center into which the electron cascades is not predictable, but has a probability following the quantum interpretation.

Within the geometrical theory, these statistics are not generated from a fundamental supposition but are a result of the system evolution: (1) The collapse of the diffracted electron wave function proceeds in a finite time according to the limits of relativity theory. It is not instantaneous. (2) The forces which induce the

process come from the advanced fields of the particles that absorb the emitted photons. The geometrical theories require an explicit well-defined construction for these forces (of radiative reaction), otherwise equivalence cannot be guaranteed. (3) The radiation is unique to the final electron state. The equations are time reversible and deterministic. The possible final states of the electron are complete and orthogonal. Unitarity demands that the final photon states map to the final electron state. (4) There is no fundamental statistical mechanism. The different results must be taken as due to differences in the configuration of the system of particles that absorb the photons as they are emitted. Experiments with radiation in cavities or those that verify correlations make any other conclusion hard to justify.

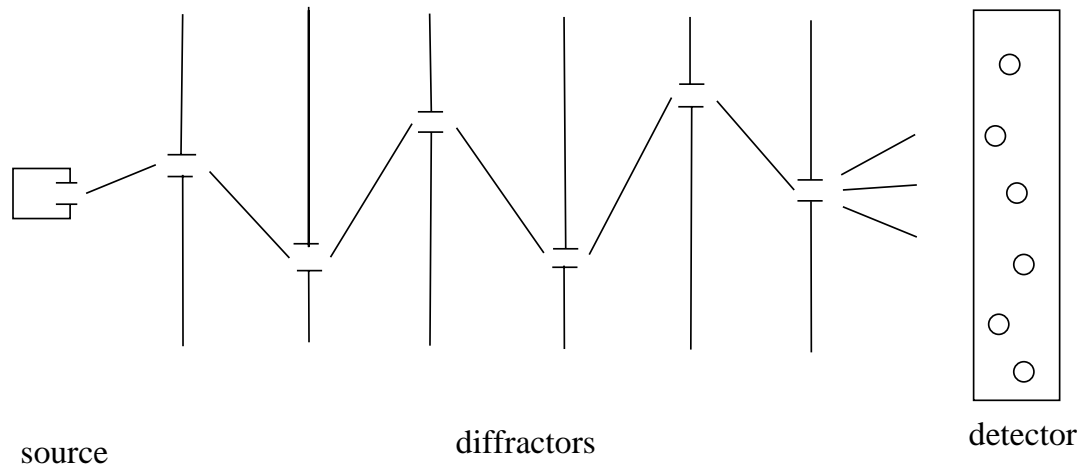


Figure 5: Successive diffractions appear to refine the particle position. In practice, this is always accompanied by interactions and radiation.

Suppose that the above experiment is extended by placing an additional series of slits between the source and detector. It is often argued that such successive refinement can identify the point position of a particle. Each slit diffracts the particle further, and by integrating backwards along the probability current, a particular trajectory within the initial emitted wave function can be defined to any limit of precision. It is sometimes argued that this shows that a particle travels on a specific trajectory. No interpretation of this type is demanded by geometrical theory. In fact, the backwards projected refinement of the original trajectory is invalid because radiative interactions are part of the selection process at each slit.

In general it is not experimentally possible to identify a specific geometrical trajectory. To describe particles as individual but enumerable waves is sufficient. The wave function, as a collection of lines of probability flow, and not any single one-dimensional trajectory is the dynamical structure in the geometry. A proposed experimental separation of a single electron trajectory within the total current flow is not supported by the geometrical constructions of this discussion and can be

However, without a quantum-gravitational theory, the internal consistency of this construction is uncertain. In general relativity it is this quantum property that permits coordinate measurement. It could not be more fundamental. The assignment of the structure of space-time by using material objects appears to be self-consistent. Tests objects rotate without change of length and the conformal stability of the observer's metric is confirmed. Can we be sure that this is always true?

Figure 7: The fundamental local scale comes from the particles that make up a reference object. The basic quantity is the Compton wavelength of the electron.

Philosophy tells us that the dimensionality of space and not just the gauge is inferred by the structure and interactions of objects or particles. In effect, the space-time coordinates are assigned by the properties of the solutions of the quantum wave equation.

In later sections five and eight dimensional fundamental spaces will be introduced to describe scalar and spinor quantum theories respectively. A specific association of geometry with physical particles is implied in each case. It is expected that a higher number of dimensions will be able to provide 3+1 effective parameters for space-time. Different metrical structures, correctly interrelated, are the essential tool.

8 Absolute Equivalence

The equivalence of gravitational and inertial mass has a long history. Modern experiments are designed to systematically test the relative contributions of different types of mass-energy. The integrated effects of electrodynamic, quantum, weak, or strong forces are tested against each other with a sensitive balance.

For general relativity, exact equivalence is expressed by the existence of a local frame. All experiments show that the equivalent of one type of mass is indistinguishable from any other. In the general case, the coordinates that would be needed to

Figure 8: Equivalence is tested by balancing different materials in combined gravitational and accelerational fields.

display the inter-transformation of forces are not experimentally accessible. This is a limitation of the four dimensional geometry. A complete mathematical explanation of equivalence must go beyond general relativity.

Ultimate verification is critical. Only if equivalence is exactly satisfied can it be expected that the various forces will interchange by geometrical transformation. The experiments cannot prove such a conclusion, but only set upper limits on deviations. Current results are reasonably precise, and deviations are not expected; hence, exact absolute equivalence is assumed. It is supposed here that any type of force can be transformed into any other type, at least internally. The limits set by the restrictions of experimental observation may conceal additional symmetries of the internal mathematics. Verification must depend on the development of a specific geometrical thesis.

It is essential to understand how to combine geometrical equivalence with quantum mechanics. Statistical effects are postponed, so that suitable mathematical conditions are applied to the fields before the generation of any probabilities. Equivalence is enforced exactly, and without random statistical contributions. The deterministic interpretation, as discussed earlier, allows this point of view.

Figure 9: A quantum particle must be represented by all of the lines of its probability current. A local inertial frame only applies to the local motion of a single quantum particle. Each particle must have its own set of local frames.

There are a number of mathematical statements that are used as conditions of equivalence. Not all of these can be adapted to the quantum case. Those formulated in terms of point classical particles are only usable in the classical limit. Fortunately, even though the particle may extend over a macroscopic region, a definition based on the notion of a local frame can be adapted. In this way the localization of the particle is not constrained. The region to which the local approximation applies must be reduced to the atomic scale. The characteristic quantum motion is taken as the probability current and, the proper frame must be sufficiently small that the flow lines show only infinitesimal acceleration. The principle of equivalence is applied to the entire probability current as it is divided into local regions of approximation.

Quantum-gravitational equivalence is more subtle than the equivalent construction in classical physics. It applies to one particle at a time, and not necessarily to a physical region of the observer's space-time. As an example, a convergent particle wave could be from either a diffraction event or from the converging force of a gravitational field. The equivalence of derivatives in the wave equation to those in the Christoffel symbols is essential. The identification as to which ones are operating requires knowledge of the cause of the convergence. It cannot be elucidated by a study of the wave alone. Quantum interference and gravitational forces are brought together.

9 Elementary Issues for Five Dimensions

The five dimensional theory is developed from physical space time. The proper-time interval equation $d\tau^2 = g_{\mu\nu}dx^\mu dx^\nu$ is rewritten formally as:

$$\dot{\gamma}_{mn}dx^m dx^n = 0 \quad (10)$$

where $dx^5 \equiv d\tau$, $\dot{\gamma}_{55} = -1$, $\dot{\gamma}_{5\mu} = 0$ and $\dot{g}_{\mu\nu} = \dot{\gamma}_{\mu\nu}$. (Lower case Greek indices are summed over four values and lower case Latin indices are summed over five.)

This formal quantity is conveniently called the neutral observer's metric. It contains available information about the local gravitational and inertial field. In addition, it is a reference point for more complicated structures that are basic to the discussion of quantum or electromagnetic effects. A number of classical studies demonstrate that the introduction of off-diagonal terms in this formal five-metric gives a correct description of electromagnetic effects⁴. Klein argues, in addition, for quantum effects⁵.

This construction implies a particular relationship between the five dimensional system and the four dimensional system. Four of the five coordinates are distinguished. This choice must eventually be justified by mechanical predictions of particle motion. It is to be shown, once interactions are developed, that this space-time is a correct parameterization for the particle motion.

It is known generally that electromagnetic or quantum effects occur when $\gamma_{\mu 5} \neq 0$. Modern quantum theory demonstrates considerable complexity beyond what can be described by a single five metric. Two quantum particles may have wave functions which overlap. The correct description requires distinct electromagnetic fields for each. It is known that an electron, described as a matter field, is not affected by its own emitted electrodynamic field. Presumably, the gravitational interaction will be similar. Two non-localized gravitating particles will, in general overlap and each will show forces that vary over the span of the wave function. The neutral observers' metric represents only the sum-average of external gravitational effects and neglects small particle to particle forces. (The concept of a universal metric with motion of singularities must be relinquished when the particles are no longer point-like.) Each particle must have, in addition to a wave function, some sort of local field representation. For these reasons, a five metric γ_{mn} is chosen.

$$\gamma_{mn} = \begin{pmatrix} g_{\mu\nu} - \mathcal{A}_\mu \mathcal{A}_\nu & \mathcal{A}_\mu \\ \mathcal{A}_\nu & -1 \end{pmatrix} \quad (11)$$

⁴See T. Kaluza, Sitz. Preuss. Akad. Wiss. Berlin, Phys.-Math. K. 966(1921), translated in *An introduction to Kaluza-Klein theories* R. C. Lee (ed.) (World Scientific, Singapore, 1984, p1), *Unified field theories of more than four dimensions*, V. De Sabbata (ed.), (World Scientific, Singapore, 1983, p427).

⁵See O. Klein, "Quantentheorie und fünfdimensionale Relativitätstheorie" Z. Phys. **37**, 895(1926), translated in *An Introduction to Kaluza-Klein theories*, R. C. Lee (ed.) World Scientific, Singapore, 1984, p10; *Unified field theories of more than four dimensions* V. Sabbata (ed.) World scientific, Singapore, 1983, p.434.

The terms \mathcal{A}_μ are the vector potential of the electromagnetic field for the geodesic motion of that particle motion, normalized to velocity units. For quantum-gravitation effects an individual four metric $g_{\mu\nu}$ must also be used. Most observations are not sensitive to this difference and it is often useful to take a universal four metric.

As each particle has a vector potential, each must have its own electromagnetic gauge. For a geometrical theory, a fixed gauge choice is made and is specified, in the classical approximation, by setting the action to zero, $S = 0$, or, in the quantum case, by constraining the wave function to be real and phase free. The simplification is important as it allows an efficient use of geometrical resources. If an explicit phase or action is needed, it can be regenerated by the reverse gauge change⁶. This geometrical gauge is assumed throughout.

The classical geodesics of the five metric have been studied.⁷ A preferred geodesic, in this gauge is

$$\frac{dx^\mu}{ds} = g^{\mu\nu} \mathcal{A}_\nu \quad (12)$$

and is known to represent correctly the Lorentz force plus gravity. This definition applies to the quantum case where, because of this gauge choice, the probability current is everywhere tangent to the geodesics.

Five dimensional coordinate transformations are allowed in principle. To avoid a theory that cannot be interpreted by the neutral observer, transformations that mix the fifth coordinate into space-time are suppressed. The local Lorentz transformations form a subgroup. There remain transformations that introduce space-time dependencies into the fifth coordinate. Writing $\tau' = x^5$, these are of the form

$$x^{\mu'} = x^\mu \quad (13)$$

$$\tau' = \Phi(x^\mu) + \tau \quad (14)$$

In classical theory, this is a cut transformation⁸. The terminology may be retained, but the interpretation is modified for quantum systems. Whenever \mathcal{A}_μ is fixed gauge, a cut transformation modifies the geodesics. In the classical limit, the motion is constrained by the Hamilton-Jacobi equation, leading to the specification of new initial conditions for fixed external gravitational and electromagnetic fields. In the quantum case, a variant of the Klein-Gordon equation is to the point, and the resulting coordinate changes include the description of interference. That is, it provides a mechanism for particle current deflections in the absence of the classically defined force fields.

⁶See the archived preprint, D. C. Galehouse, "Quantum geodesics", xxx.lanl.gov gr-qc/9512034.

⁷See V. Fock, Z. Phys. **39**,226(1926); E. Leibowitz, N. Rosen, Gen. Rel. and Grav. **4**,449(1973); G. Stephenson, C. W. Kilmister, Nuovo Cimento **10**,230(1953).

⁸As defined in A. Einstein, P. Bergmann, "On a Generalization of Kaluza's Theory of electricity", Ann. Math. **39**, 683(1938)

The objective is to find a set of differential equations that define the motion and evolution of particles with their associated wave functions, metrics and electromagnetic fields. A five covariant theory may yet be allowed, so, it is natural to look for differential invariants as they come from the Riemann tensor in five dimensions. Some of these quantities may not depend explicitly on τ , and may include quantum field equations along with interactions.

10 Essential Results in Five Dimensions

First, it is known that a covariant quantum field equation can be generated from the five metric⁹ and references contained therein. With $\lambda = 1$ and $\chi = 1$, the curvature scalar, $\Theta \equiv \Theta(\gamma_{\mu\nu}) \equiv \Theta(\gamma_{\mu\nu}(\omega))$ becomes, upon being set equal to zero in a field free region,

$$(\partial)_{x^m}^2 \Psi \equiv \left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial \tau^2} \right) \Psi = 0 \quad (15)$$

Where the linear wave function Ψ is equal to $\omega^{3/4}$. Adding the condition

$$\frac{\partial \Psi}{\partial \tau} = im \quad (16)$$

produces a Klein-Gordon equation for mass m . This integrates quantum theory into the geometry, the important property being that the result is independent of the coordinate system. The additional terms in \mathcal{A}_μ and $g_{\mu\nu}$ that represent external fields appear correctly when the full scalar curvature is calculated. The expansion produces corrections to the motion when fields are very high, perhaps affecting photo-production or the analogous gravitational process.

Suppose that an interacting system contains a set of particle fields expressed one to one by five metrics of the form of equation (11). With the geometrical fixed gauge restriction, new conformal factors can be included. The geodesic motion is unchanged if the metric is written

$$\gamma_{mn}(\omega, \lambda, \chi) = \omega \begin{pmatrix} \lambda g_{\mu\nu} - \chi^2 \mathcal{A}_\mu \mathcal{A}_\nu & \chi \mathcal{A}_\mu \\ \chi \mathcal{A}_\nu & -1 \end{pmatrix}. \quad (17)$$

The geodesics

$$\frac{dx^\mu}{ds} = \omega g^{\mu\nu} \chi \mathcal{A}_\nu \quad (18)$$

are the same and remain tangent to the probability current. Changes in $\lambda(x^\mu)$, $\chi(x^m)$, and $\omega(x^m)$ produce a new tensor but represent the same congruence of trajectories.

The assertion is that the conformal factors λ , χ , and ω may connect parametrically with external interactions. They effect the causative forces of a given motion.

⁹See the archived preprint, D. C. Galehouse, "Quantum-Conformal Field Theory", xxx.lanl.gov gr-qc/9512035

An active change in a conformal factor might convert an observed electromagnetic force into a gravitational or quantum force. In this way, the usual Lorentz equivalence of electric and magnetic source currents extends to gravity and quantum mechanics.

By definition, an electric or magnetic force is precisely what would be found in the classical limit, there being no other way to separate quantum effects. The tensor, (11), without the conformal transformations or the associated curvature, must serve as a reference to define what is meant experimentally by an observed fields.

The factors λ , χ , and ω may generate the effective force. The simplest mechanism is to assume that external effects derive from the apparent curvature caused by implicit but unidentified conformal factors. Setting the five Ricci tensor to zero

$$0 = \Theta^{ij}(\gamma_{mn}(\omega, \lambda, \chi)) \quad (19)$$

produces a constraint between the conformal parameters and the original five metric, (11). In analogy with the usual gravitational field equation, additional curvature can be added by placing other quantities on the left of this equation, but a certain minimum interaction can be developed directly. Equation (19) is recast into the form.

$$\Theta^{ij}(\gamma_{mn}(1, 1, 1)) = T^{ij}(\omega, \lambda, \chi). \quad (20)$$

where $\gamma_{mn}(1, 1, 1)$ is the usual five metric and T^{ij} is to be found as a function of ω , λ , and χ , and eventually, the physical source currents.

The equation for $\Theta^{ij}(\gamma_{mn}(\omega, \lambda, \chi))$ can be inverted to give explicitly the dependency of $\Theta^{ij}(\gamma_{mn}(1, 1, 1))$ on the conformal parameters. The calculation involving χ and λ is straightforward but algebraically very involved. It shows that that the source terms generated for T^{ij} from λ and χ always depend essentially on the motion of the test particle. This fails in the classical limit because T^{ij} must depend on the source particle alone. Thus, a use of χ and λ has a more complicated character, possibly involving some sort of geometrical self-interaction. It is not helpful for identifying the external dependency. The quantities χ and λ are therefore dropped.

The remaining factor, ω , can be Taylor expanded at a point. The zero order term does not contribute and the space-time part of the first order term can be removed by a five space conformal gauge transformation. The essential remaining portion is $\omega = B_{nm}x^m x^n$. (Higher order terms from the expansion do not contribute because the Ricci tensor is second order.) This is substituted into the standard calculation for the conformal contribution. The available tensors that might be used to make up B_{nm} are either the particle five metric or the metric of the neutral observer. A somewhat contrived but revealing assumption is

$$B_{ij} = \frac{|\psi|^2 f}{2 - \frac{e^2}{m^2} A^2} \left[\begin{pmatrix} \dot{g}_{\mu\nu} & \\ & -1 \end{pmatrix} - \frac{7}{8} \begin{pmatrix} \dot{g}_{\mu\nu} - A_\mu A_\nu & A_\mu \\ A_\nu & -1 \end{pmatrix} \right]$$

where f, χ , and λ are numerical parameters that can be adjusted to fit known interaction constants. This becomes, as expressed in four dimensional notation,

$$R^{\alpha\beta} = 8\pi\kappa[F^\alpha{}_\mu F^{\mu\beta} + m|\psi|^2 \frac{e^2}{m^2} A^\alpha A^\beta + m|\psi|^2 \frac{1 - (e^2/m^2)A^2}{2 - (e^2/m^2)A^2} g^{\alpha\beta}]. \quad (21)$$

and at the same time

$$F^{\beta\mu}|_\mu = 4\pi\alpha|\psi|^2 A^\beta. \quad (22)$$

Notwithstanding some of the simplifications made in the derivation, this result agrees with known observations and accepted theory. Equation (21) represents the Einstein field equation for the metric of a single quantum particle. The electromagnetic energy density source term is standard as in the Einstein-Maxwell theory. The second contribution is the quantum coherent representation of the matter term for a single source particle. (The vector potential in standard units must be multiplied by the factor e/m to become a standard velocity.) It is easy to see that this reduces to the incoherent matter term when quantum corrections to the effective velocity are unimportant and an integral over particles changes the quantum density to a particle density. The third term is new, very small, and can have either sign. The coefficient is of order $\hbar\kappa$ and because the numerator of the fraction goes to zero in the classical limit, the net effect is further reduced by averaging. This last term, being proportional to $g_{\mu\nu}$, is a quantum correction to the pressure and, when averaged, contributes to an effective cosmological constant.

The second equation is precisely Maxwell's inhomogeneous equation for a quantum source particle. No other terms appear on the right side, to any order. This detail is important because corrections would likely have been observed by precision tests of quantum electrodynamics. Here, as above, the source for the electromagnetic field is the probability current flow density and not any statistical sum over classical particles. The equivalence between the gravitational and electromagnetic fields requires a similar source structure for both. The quantum properties must be analogous. In general, the fields are not separable except for the assumption that the system is viewed by a particular neutral observer.

The condition of inferred dimensionality is satisfied. Explicit reference to the fifth coordinate is absent, and the resulting system predicts quantum dynamical interactions that are in appearance four-dimensional. The assumption of a particular subspace for space-time is confirmed. The concept of the mechanical gauge is usable and has a natural scale size depending on the separation constant, m , of the field equation. In fact, a universal mechanical gauge is realized by choosing the same type of particle at each point and relating the scale of each mass to the fifth coordinate. (Scale changes in τ thus cause proportional changes in the local mass of all particles and are unobservable.) Moreover, the concept of mass comes from the geometrical structure. Experimentally, observed particles having other masses also have additional interactions. If these forces are to be expressed within the paradigm

of absolute equivalence, a higher geometry must be used. Such a system may provide characteristic values, perhaps even separation constants, that would give mass spectra.

The real strength of the geometrical covariance is that it describes, without inconsistencies, relativistic fields that represent accelerated motion. There are many possible applications, from massive compact astrophysical objects to elementary particles. A theory that is only Lorentz covariant is surely inadequate to treat the details. The curvilinear approach has the capability to describe these complexities and, as the mathematical tools are developed, to provide needed insight.

11 Time Symmetry and Field Quantization

Modern theories of interaction use advanced potentials. It has been shown that an assumption of time symmetry for the radiated field of a charged particle gives results identical to the assumption of retardation. The time symmetric formalism has some advantages. The most important is that a formal second quantization of the electromagnetic field is not required. True free fields are not needed since the equivalent physical mechanism is supplied by real particles that make up the radiative milieu.

The ontology of a distant fundamental absorber remains uncertain. In the electromagnetic case, the density of available transitions by real particles seem sufficient and prevents an experimental demonstration of either reduced absorption or of absorption at infinity. It may be that the concept of an abstract absorber at infinity should be dropped. If this is done, the vacuum becomes the sum effect of available quantum transitions that are null connected to the event at hand. This approach attributes the forces of radiative reaction to the specific particles that are absorbing the radiation. In effect, if spontaneous radiation is to occur, all particles that might receive that radiation, including those at infinity must be included in the calculation from the beginning¹⁰. Field quantization is not needed because each photon, that would be part of the free field, is actually the field of a particle that is participating in the interaction. A spontaneous emission event, mediated by a large number of interacting particles, might be harder to calculate but the result is the same as for a model that attributes the fields of those particles to free field modes¹¹.

A form of mathematical determinism is obtained. A system defined by the completion of all particles that are light like interconnected can be deterministic. This condition is strong, and the classical concept of an isolated system fails, not just sometimes but universally. Indeterminism must be part of our observations, not from any intrinsic supposition, but because there is always an outside interaction,

¹⁰An early discussion is by M. Reminger, *Z. Phys.* **158**, 417(1960).

¹¹See D. Leiter, *Am. J. Phys.* **38**, 207(1970), *Int. J. Theor. Phys.* **3**,205(1970), P. C. W. Davies, *Proc. Cambridge Philos. Soc.* **68**, 751(1970), *J. Phys. A* **4**, 836(1971);**5**, 1025(1972), and a review article D. T. Pegg, *Rep. Prog. Phys.* **38**,1339(1975).

from other particles. The cat paradox is simplified. The cat is isolated only in the classical sense. Events in the box cannot be predicted because isolation from the universe is incomplete. A practical physical determinism is not attained.

Because of the gravitational non-linearities, a fundamental assumption of retardation is physically distinct from time symmetry. The non-linearities affect the result (independently of the issues of energy conservation during absorber interactions) and the addition of boundary conditions generated at infinity affects the motion in the test region. Simplicity of structure and the similarity of gravitational and electromagnetic fields suggest that fundamental fields are always time-symmetric and that radiative reaction forces, electromagnetic or gravitational are always due to the advanced fields of interacting particles. There is no proof.

The five dimensional construction is not capable of representing primitive uncharged particles. Neutral objects must be built up. Assuming appropriate boundary conditions for the gravitational field, gravitational radiation can be calculated, at least in principle, without troubles from singularities. The question of retardation must be resolved by experiment, but further complexities arise because any of the particles can radiate electro-magnetically. The gravitational effects may be separable because of collective motion, but the extent of the cancellation and the residuals have not been studied with respect to the issue of time symmetry. An unambiguous separation may not always occur. The issue remains somewhat unsatisfactory. If it turns out that the separation into gravitational and electromagnetic radiation is unique and well defined under all physical conditions, then the reduced strength of the gravitational interaction at a distance may not allow for the formation of completely retarded fields. In the absence of a universal absorber, advanced gravitational interactions may be indicated. Experimental results are yet to be obtained on this point.

With the denial of a formal process of first quantization, higher 'quantizations' and such associated structures as equal time commutators are no longer well-motivated. Raising and lowering operators have always been a form of particle bookkeeping and have nothing to do with the classical to quantum transition. In the case of quantum field theory, the assumption of a classical Lagrangian for field operators followed by a quantization step is fraught with danger, particularly in regard to covariance of the result. Equal time commutators serve simply to adjoin quantum correction terms to the equations. They insure that the essential terms are correctly assigned to each individual field particle in turn. For a gravitational five theory, the process is equivalent to incrementing the number of metrics, or analogous covariant geometrical structures as the particle number increases. As new interactions are added, geometrical elements become larger and counting appears to be simpler than trying to define the equal-time commutators in a non-linear system.

To insure covariance for other interactions, higher Riemann tensors may be necessary¹². For a given level of geometrical complexity, only certain interactions are

¹²See O. Veblen, *Invariants of Quadratic Differential Forms*, (Cambridge U. 1933).

allowed. The constraints are quite severe. The usual perturbative approach of quantum field theory assumes arbitrary fields and then looks to constrain and quantize. The result must be made covariant, and with the quantization problem clouding the way, finding a successful combination of fields may be unreasonably difficult.

12 Spin

A consistent study of spin requires awareness of the interactions that go with it. Because spin has an established basis in geometry the needed structure should be available. When the Dirac formalism is included, a new interaction becomes important. Knowing that electron-neutrino scattering can be represented with the Dirac theory, insures a connection to the weak interaction. A correct theory of spin must satisfy equivalence, implying in turn, that the weak interactions are integrated into the geometry. A half theory, that adds forces phenomenologically, cannot satisfy equivalence and so must eventually be replaced.

The imperative for the extension to spinors is simply that there are no known stable massive bosons. All decay into other particles which have, eventually, spin $1/2$. The group properties of the Lorentz transformations and the modern studies of spin in general relativity suggest that spin is a fundamental part of physics, and space-time. A study of Cartan's¹³ book supports this view, and interest has continued unabated since the discovery of the relativistic equation of the electron by Dirac. The conformal five dimensional theory should be extended to describe a physical electron. None of the fields so far developed have spinor transformation properties; however, the deeper properties of the irreducible group representation must be there. It is proposed that the spin structure is hidden in geometry. The question of how this might be found occupies the remainder of this chapter.

The practical evidence appears on all levels. Simplest is probably the observation that the most primitive geometry comes from mechanical experience and depends on objects held uncollapsed by the Pauli forces. The kinematics are correctly represented by the scalar theory but the dynamical repulsion of paired electrons requires spin states and spin coupling. The constancy of the length of a rod, while it is being rotated, depends in a practical way on spin antisymmetry.

Spin one systems can be constructed from spin one half systems. This may be analogous to the five dimensional description in which the spin one photons and spin two gravitons are part of a single five-scalar. Should there be an analogous structure in which massless spin one-half solutions are part of a similar conformal structure with photons and gravitons?

The antisymmetrization of electrons needs a geometry. Consider as shown in figure (10) a pair production event, after which, the positron annihilates with an outside electron. While each electron must not be affected by its own source currents, it

¹³See E. Cartan, *The Theory of Spinors*, M.I.T. Press, Cambridge, Mass. 1966.

Figure 10: The positron from an e-p pair annihilates smoothly with another electron. The electrons must be antisymmetric with respect to each other.

must interact with the currents of the other, even though they eventually connect through the positron. The enigma has not been resolved in the context of a geometrical theory. Antisymmetrization and the associated indistinguishability seem important. This and the ubiquitous presence of spin $1/2$ are taken as symptoms of a deeper geometry.

Of peripheral interest is an explicit resolution of spin correlations¹⁴. In principle, the essential behavior is demonstrated by a pair of spin one particles. This case can be addressed with the five dimensional system. Two pair of oppositely charged spin zero bosons are combined to form two spin one particles. (These may exist transiently, but the formal issues are present in any case.) As composite particles, they can interact electrostatically and demonstrate the usual behavior with respect to correlations during state reduction. The five theory produces the accepted result in a deterministic setting. The forces of radiative reaction (or the advanced

¹⁴A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. **47**,777(1935).

forces, as you will) plus the distributed, overlapping nature of coupled spin systems is sufficient. To be certain of the equivalent construction for spin one-half requires a fundamental geometrical theory. There is no reason to anticipate difficulty. The usual electromagnetic coupling will have the required properties (as will any relativistically covariant interaction) and a pair of relativistic particles experience the same types of interactive forces whether they are spinning or not.

More esoteric are phenomena that depend on combinations of spin and general relativity. Consider a polarized electron sitting at the center of a Thirring sphere. A change in rotation rate of the sphere will drag the inertial frame in the neighborhood of the electron without interaction. A valid representation of the final electron state is enigmatic.

13 Pauli-Dirac Theory

The most elementary starting point is to reconsider the discussion of Pauli and others¹⁵ in which the five anti-commuting Dirac matrices are each associated with one of the five coordinate directions. Let

$$\dot{\gamma}^0, \dot{\gamma}^1, \dots, \dot{\gamma}^4 \equiv \dot{\gamma}_{AB} \quad (23)$$

for $m = 0, \dots, 4$ and $A, B = 1, \dots, 4$ such that

$$\mathbf{1}\dot{\gamma}^{mn} \equiv \mathbf{1} \begin{pmatrix} \dot{g}^{\mu\nu} & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{2}\{\dot{\gamma}^m, \dot{\gamma}^n\} \equiv \frac{1}{2}(\gamma^m\gamma^n + \gamma^n\gamma^m) \quad (24)$$

It is convenient to use the standard basis for γ^μ , $\mu = 1 \dots 4$ and to let $\dot{\gamma}^4 = i\gamma^5$, with $\mathbf{1}$ the unit matrix. For the particle metric (as used to represent the fields of an individual particle, say the electron.)

$$\mathbf{1}\gamma^{mn} = \mathbf{1} \begin{pmatrix} \dot{g}^{\mu\nu} & \mathcal{A}^\mu \\ \mathcal{A}^\nu & \mathcal{A}^\tau \mathcal{A}^\rho \dot{g}_{\tau\rho} - 1 \end{pmatrix} = \frac{1}{2}\{\gamma^m, \gamma^n\} \quad (25)$$

where $\gamma^\mu = \dot{\gamma}^\mu$ and $\gamma^4 = \dot{\gamma}^4 - \mathcal{A}_\mu \dot{\gamma}^\mu$

¹⁵See the early work by W. Pauli "Über die Formulierung der Naturgesetze mit fünf Homogenen Koordinaten I", Ann. Phys. **18** (1933) 305–336, W. Pauli "Über die Formulierung der Naturgesetze mit fünf Homogenen Koordinaten II", Ann. Phys. **18** (1933) 337–372, W. Pauli, J. Solomon "La Theorie Unitaire D'Einstein et Mayer et les Equation de Dirac" J. Phys. Rad. **3** (1932) 452–463, W. Pauli, J. Solomon "La Theorie Unitaire D'Einstein et Mayer et les Equation de Dirac" J. Phys. Rad. **3** (1932) 582–589, V. Fock, "L'Equation d'Onde de Dirac et la Geometrie de Riemann", J. Phys. Rad. **10** (1929) 392–405, V. Fock, D. Iwanenko, "Über eine mögliche geometrische Deutung der relativistischen quantentheorie.", bf 64 (1929) 798–802, V. Fock "Geometrisierung der Diraschen Theorie des Electrons", Z. Phys. **57** (1929) 261–277, B. L. van der Waerden "Spinoranalyse", Gott. Nachr. (1929) 100–109, D. Iwanenko "Bemerkung über quantenmechanische Geschwindigkeit" Z. Physik **55** No. 2 (1929) 141–144.

The standard Dirac equation

$$i\hbar\gamma^\mu\frac{\partial}{\partial x^\mu}\psi - mc\psi = 0 \quad (26)$$

can be rewritten in the five dimensional notation. Define a similarity transformation by $S = \frac{1}{\sqrt{2}}(\mathbf{1} + \gamma^4)$, then left multiply by $S\dot{\gamma}^4$, right multiply by $e^{i\frac{mc}{\hbar}x^4}$, and insert the pair $S^{-1}S$.

$$i\hbar(S\gamma^4\gamma^\mu S^{-1}\frac{\partial}{\partial x^\mu}S\psi e^{i\frac{mc}{\hbar}x^4} - mcS\gamma^4S^{-1}\psi e^{i\frac{mc}{\hbar}x^4}) = 0 \quad (27)$$

Setting $\Psi = S\psi e^{i\frac{mc}{\hbar}x^4}$ and noting that $S\gamma^4S^{-1} = \gamma^4$ and $S\gamma^4\gamma^\mu S^{-1} = \gamma^\mu$ gives

$$\gamma^m\frac{\partial}{\partial x^m}\Psi = 0. \quad (28)$$

An interesting question is how to connect the conformal part of the Dirac spinor wave function Ψ with the five conformal factor ω . That is, if $\gamma^{ij} \rightarrow \omega\gamma^{ij}$, is this equivalent to $\gamma^i \rightarrow \omega^{1/2}\gamma^i$ or $\Psi \rightarrow \omega^{1/2}\Psi$? Certainly, there are conformal contributions to the curvature and other complexities as well, but because of the conformal flatness of the five theory, a construction of some kind should be possible. The conformal substructure of the Dirac theory is at issue. A possible immediate application is a fundamental understanding of the interaction of spin one-half particles with the gravitational field.

14 Spinor Coordinate Systems

More radical is to search for a geometrical system that has spinors as a natural tangent space. Classically, the choice of local frame is not unique because space rotations about the particle center are degenerate. For a real electron, the spin breaks the symmetry. It is conventional to assign a spinor basis space in place of a local Lorenz frame. But, to develop a fundamental geometry of spin, the coordinates themselves should provide the intrinsic local orientation. In four and five dimensions the tangent spaces are natural. There should be a similar coordinate space for spinors.

In response to this possibility, it is conjectured that a spinor tangent space, attached to one particle, can be constructed naturally from an augmented base space. Let there be four complex valued coordinates $(\xi^1 \dots \xi^4) \equiv \xi^A \in \mathbf{C}$ which are to be related to the real five space coordinates $(x^0 \dots x^4) \equiv x^m$. It may be helpful that the five space is flat, avoiding some of the usual difficulties with holonomy.

A number of transformations are possible, but not all have suitable properties. The following construction is under study. Let the complex coordinates ξ^A be written

$$\xi^A = \xi_r^A + i\xi_i^A \quad (29)$$

with ξ_r^A, ξ_i^A real. Define a coordinate relation by

$$x^m = \frac{1}{2} \gamma_{AB}^m \bar{\xi}^A \xi^B. \quad (30)$$

where $\bar{\xi}^A$ is the complex conjugate. A five dimensional real space is generated by a restriction from the four dimensional complex space. The particle five metric, and from that the observer's metric, obtain their scaling and conformal structure from the spinor mapping functions. Symmetry requires this. The inferred metrical structure of the ξ space is quite different and can have none of the information contained in the mapping. Elucidation requires understanding how differential equations, natural to either space might be related.

Further characteristics may be enlightening: (1) Translations are allowed in ξ space and leave the local bundle structure intact. This is also true for \mathbf{R}^5 . (2) Apparently, to the Klein-Gordon equation $(\partial^2)_x \Psi = 0$ in five dimensional real space corresponds some characteristic equation in the complex space. Perhaps it is of the form $(\partial^2)_\xi (\partial^2)_\xi \Psi = 0$ or $(\partial^2)_\xi \Psi = 0$ where $(\partial^2)_\xi$ represents an eight dimensional d'Alembertian in the ξ_i^A and ξ_r^A . The essential point is whether the Klein Gordon equation in five space can be written in terms of a suitable differential invariant in spinor derivatives. Apparently, there are characteristic solutions on \mathbf{C}^4 that map to standard quantum wave functions on \mathbf{R}^5 . (3) The conformal structure of interactions in 5-space is transferred to the complex 4-space and induces a quantum structure with the required gravito-electromagnetic interactions in place. (4) Also, it seems that the relevant interactions, as induced by the second order derivatives of the conformal factor in \mathbf{R}^5 are augmented by other derivatives in \mathbf{C}^4 . Thus, additional conformal variations having nonzero derivatives for odd powers of ξ^A may not reduce to an expansion in derivatives by x^m . These can be neither electromagnetic, gravitational, nor quantum. They are conjectured to be effects of weak interactions. The inferred electron-neutrino scattering should be analogous to the conformal description of electron-photon and electron-graviton effects. (5) The five metric as an indefinite quadratic form is generated from the the complex coordinate space, \mathbf{C}^4 , by the properties of the gamma matrices. A mechanical gauge then comes from the dynamical properties of particular solutions of equations on \mathbf{C}^4 based on the symmetry of coordinate combinations as specified by the γ 's.

A number of conjectures remain indeterminate. (1) Is it possible to represent the Dirac wave function as a unit vector in \mathbf{C}^4 multiplied by a conformal factor? (2) What, in detail, is the differential geometry of this scheme? (3) Is the phenomenology correct? (4) Can all known properties of the electron be described in this way? (5) What additional structures may be needed? (6) Is it correct to use a general complex nonsingular transformations for the ξ coordinate system? (7) Are there other analogous coordinate spaces and transformation systems that might apply to higher particles?

15 Summary

Presented here are new results, that describe a closed form theory of quantum particles moving in combined gravitational and electromagnetic fields. Relinquishment of quantization abolishes the need for a classical theory. Exclusion of constructions that depend on discrete classical particles allows a fully wave oriented development. A dynamics based on conformal effects permits a mathematically simple scheme. Equivalence is presumed for all interactions and particles. Four dimensional space-time comes out of higher dimensional theory by identifying properties of the particles as they appear in the geometry.

Certain ideas enable the introduction of spinors. Spin has always been geometrical and the Dirac equation needs a curvilinear geometrical foundation. It seems natural to associate each one of the five anti-commuting Dirac matrices with one of the five coordinates. Explicit introduction of spinor coordinates, particle by particle, may be possible. In any case, a geometrical structure with interactions is expected

The immediate goal, as a problem in physics, is to understand electrons and their interactions. The mathematical goal is to understand how to handle particle geometries so that more complicated and realistic systems might be accessible.

16 Acknowledgement

This article began as a talk given at the 6th International Conference on Clifford Algebras in Cookville, Tenn., U.S.A. The first written version is posted on the archives as math-ph/021077.

Figure 11: The Dirac-Thirring paradox. A spin one-half electron in the up state is placed inside a large mass shell. If the shell rotates with the axis along the spin direction, the local inertial frame is dragged along too. This should reduce the electron's angular momentum. A superposition of the up and down state is needed but the relative phase of the spin states cannot be determined since there is no mechanism to break the angular degeneracy. If the shell rotates against the spin direction, the represented angular momentum should increase. No higher rotation state is available. Unitarity fails either way.

labelfig:dizzy

Figure 12: Electrons have spinor type local tangent frames. These are not naturally derivable from the usual coordinate space. A new coordinate super-manifold is needed.

Figure 13: The complex plane can be mapped stereo-graphically to the surface of a sphere in two different ways. The projection line from a point on the plane can go through either the center or the antipode. Both cases are shown together. Two spheres of radius one and two are tangent at the origin, point O . The line projection goes through the point P which is at the center of the larger sphere and at the antipode of the smaller one. The angle θ from the vertical axis to the projected point on the smaller sphere is exactly twice the angle ϕ for the larger sphere. There is a quadratic mapping from one to the other. The double valued relation of ξ to x is analogous to the required spinor map of \mathbf{R}^5 to \mathbf{C}^4 suggesting a spinor coordinate transformation.