Optimal Beamforming for Spectrum Leasing in Cognitive Radio Networks

Seyed Mohammad-Sajad Sadough, Ardalan Alizadeh

Abstract – In this paper, the primary network leases some part of its available resources to cognitive radio (CR) users, with the aim of increasing its achievable data rates. In counterpart, CR users are allowed to exploit the leased resource for their own transmission. By using distributed beamforming among CR nodes, we formulate a beamforming problem so as to optimize the parameters involved in the leasing process. Simulation results demonstrate that the proposed leasing method provides benefits for both primary and CR networks.

Keywords: Cognitive Radio, Spectrum Leasing, Cooperative Relay Networks

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_r$</td>
<td>Primary transmitter power</td>
</tr>
<tr>
<td>$f_k$</td>
<td>Vectors of the channel coefficients for $k = 1, 2$</td>
</tr>
<tr>
<td>$n_{cr}$</td>
<td>Number of cognitive nodes</td>
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<tr>
<td>$n_f$</td>
<td>Gaussian complex noise vector at CR</td>
</tr>
<tr>
<td>$w$</td>
<td>Weighting vector</td>
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<tr>
<td>$U_{cr}(\alpha, w)$</td>
<td>Utility function</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Leased time</td>
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<tr>
<td>$c$</td>
<td>Cost per unit transmission energy</td>
</tr>
<tr>
<td>$n_2$</td>
<td>Received noise from primary</td>
</tr>
<tr>
<td>$\text{SNR}_{pr}$</td>
<td>Signal-to-noise ratio at primary receiver</td>
</tr>
<tr>
<td>$P_p^{\text{non-lease}}$</td>
<td>Primary rate before leasing</td>
</tr>
<tr>
<td>$P_{\text{total}}^{\text{cr}}$</td>
<td>Total transmit power of CR relays</td>
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</table>

I. Introduction

In a widely-adopted category of cognitive radio (CR) systems [1]–[4], CR users try to have an opportunistic access to the primary users’ resources through spectrum sensing [5]. In a recent approach, referred to as spectrum leasing, primary users are aware about the presence of secondary users, and actively lease some fraction of the spectrum to these users by charging them at a certain price or service [6],[7]. Here, we propose a method for spectrum leasing based on distributed beamforming.

More precisely, we assume a common scenario where there is no direct link between the primary transmitter and receiver. Similar scheme has been provided in [8],[9] in which the primary network can communicate through the deployment of some intermediate relays.

Based on this scheme, the primary network is a pair of transmitter and receiver which can be considered as primary base station and a user in cellular networks.

In this paper, we consider some CR users distributed between primary relay nodes and the primary networks leases some part of its transmission time to CR users who acts as distributed relays for primary transmission. There is no literature in spectrum leasing approach based on spectral beamforming [6],[7] and authors considered decode-and-forward relaying scheme for optimization problem. An optimization problem is formulated for setting the optimal duration of the leasing period that satisfy the benefits for both secondary and primary networks in terms of achievable data rates.

The problem is solved by using the genetic algorithm (GA) and simulations are provided to show performance improvements achieved by using the proposed leasing method.

II. System Model

As shown in Fig. 1, the considered model consists of a pair of primary transmitter and receiver with no direct link. Therefore, $n_r$ relay nodes (referred to as primary relays) are deployed in the network for amplifying the transmitted signal and forwarding it to the primary receiver.

We assume that the weighting vector at relays has been priory calculated based on an arbitrary method such as the minimum transmit power approach provided in [8]. Moreover, we assume that a number of $n_{cr}$, ($n_{cr} > n_r$) cognitive nodes are present and distributed in the network. In practice, due to the long distance separating the primary transmitter and receiver, achieving high data rates by the primary network, requires a large number of primary relays.
Obviously, satisfying this requirement is not always possible and economically justified in wireless communication systems. For this reason, here, we consider a scheme where distributed CR nodes assist the primary network by playing the role of relays in a fraction of the time-slot dedicated for primary transmission. Hereafter, these relays are referred to as cognitive relays. As remuneration for this assistance, some part of the primary time-slot is leased by the CR network to its own secondary transmission (see the bottom of Fig. 1).

The received signal vector \( \mathbf{x} \) of size \( n_{cr} \times 1 \) at CR terminals can be written as:

\[
\mathbf{x} = \sqrt{P_1} \mathbf{f}_k \mathbf{s} + \mathbf{n}_i
\]

where \( s \) is the information symbol transmitted by the primary transmitter with power \( P_1 \), \( \mathbf{n}_i \) is the \( n_{cr} \times 1 \) Gaussian complex noise vector at CR terminals with each entry distributed as \( \mathcal{CN}(0,\sigma^2) \), and

\[
\mathbf{f}_k = \begin{bmatrix} f_{1k} & f_{2k} & \cdots & f_{n_{cr}k} \end{bmatrix}^T
\]

is the vectors of the channel coefficients for \( k = 1,2 \), and \((\cdot)^T\) stands for the transpose operator.

Note that (1) is referred to as the first relaying step. During the second step of relaying, the \( i \)-th CR node multiplies its received signal by a complex weight \( w_i \) and broadcasts it in the network.

The \( n_{cr} \times 1 \) complex vector \( \mathbf{t} \) denotes the transmitted signal from cognitive relays with

\[
\mathbf{t} = \mathbf{Wx}, \quad \text{where} \quad \mathbf{W} = \text{diag} \left[ \begin{bmatrix} w_1 & w_2^* & \cdots & w_{n_{cr}}^* \end{bmatrix} \right]
\]

with \( \text{diag} \{ \mathbf{a} \} \) being a diagonal matrix with diagonal elements equal to vector \( \mathbf{a} \). We assume that the weight vector employed at primary relays is set in advance as a result of an optimization beamforming problem solved before starting the leasing process. In other words, our aim in this paper is to set the optimal beamforming weights at cognitive relays.

The received signal at the primary receiver, denoted by \( y \), can be written as:

\[
y = \sqrt{P_2} \mathbf{W} \mathbf{f}_k \mathbf{s} + S_p + n_2 + n_3 = \sqrt{P_2} \mathbf{W} \left( \sqrt{P_1} \mathbf{f}_k \mathbf{s} + \mathbf{n}_i \right) + S_p + n_2 + n_3
\]

where \( n_2 \) is the additive white Gaussian noise in the secondary receiver with distribution \( \mathcal{CN}(0,\sigma^2) \).

\( n_3 \) is the amplified noise vectors from primary relays at the primary receiver in the second relaying phase, distributed as \( \mathcal{CN}(0,\sigma_3^2) \). Since the weighting vector of the primary relays has been considered as a fixed value before leasing, we can write the received signal from primary relays as a constant value \( S_p \). Noting that \( \mathbf{a}^T \text{diag} \{ \mathbf{b} \} = \mathbf{b}^T \text{diag} \{ \mathbf{a} \} \), we can rewrite (2) as:

\[
y = \sqrt{P_2} \mathbf{W}^H \mathbf{f}_k \mathbf{s} + \mathbf{w}^H \mathbf{F}_1 \mathbf{n}_1 + S_p + n_2 + n_3
\]

where \( \mathbf{F}_k = \text{diag}\{ \mathbf{f}_k \} \) for \( k = 1,2 \), and \( \mathbf{w} = \text{diag}\{ \mathbf{W}^H \} \), and \((\cdot)^H\) denotes Hermitian transpose. Here, \( \text{diag}\{ \mathbf{A} \} \) is a vector formed by the diagonal elements of the square matrix \( \mathbf{A} \). Then, based on (3), the received SNR in the second transmission step after performing the leasing process can be expressed as:

\[
\text{SNR}_{pr} = \frac{P_2 \mathbf{w}^H \mathbf{h}^H \mathbf{w} + |S_p|^2}{\sigma^2 + \sigma_3^2 \mathbf{w}^H \mathbf{D}_2 \mathbf{w} + \sigma_{cr}^2}
\]

where \( \mathbf{h} = \mathbf{F}_2 \mathbf{f}_2 = \mathbf{F}_2 \mathbf{f}_1 \) and \( \mathbf{D}_2 = \mathbf{F}_2 \mathbf{F}_2^H \).

Based on the system model described above, in what follows, we aim at finding the optimal weighting vector \( \mathbf{w} \) and the appropriate leasing parameters.

**III. Problem Formulation**

Parameters involved in spectrum leasing are usually set through the optimization of a utility function [6]. Assume that the fraction of time leased to the CR network is equal to \( \alpha \). Here, we define the utility function as the difference between the rate achieved by the CR network during the leased time \( \alpha \) denoted by \( R_{cr}(\alpha) \) (i.e., the advantage gained via leasing) and the consumed power during the second phase of relaying denoted by \( P_{cr}^{tot}(\mathbf{w}) \) (i.e., the cost of leasing for the CR network). Moreover, we assume that during the leased time \( \alpha \), all CR nodes involved in primary signal relaying, can communicate with the CR base station by

![Fig. 1. System architecture of the considered network](Image)
adopting the frequency-division multiple access (FDMA) scheme. The employed utility function is thus defined as:

$$\begin{align*}
U_{cr}(\alpha, w) &= R_{cr}(\alpha) - c \left( \frac{1 - \alpha}{2} \right) P_{cr}^{\text{max}}(w) = \\
&= \frac{\alpha}{n_{cr}} \sum_{i=1}^{n_{cr}} \log_2 \left( 1 + \frac{P_i |g_i|^2}{\sigma_i^2} \right) + c \left( \frac{1 - \alpha}{2} \right) w^H (P_i D_i + I) w
\end{align*}$$

(5)

where $P_i$ is the transmit power of the $i$-th CR user which is equal to the transmit power of the CR user in relaying mode, $g_i$ is the channel gain between the CR base station and the $i$-th CR user, $\sigma_i^2$ is the noise variance corrupting the transmission from the $i$-th CR to the CR base station. For making balance between parameters of utility function, a constant parameter $c$, the cost per unit transmission power dissipated at CR nodes to be upper-bounded by $P_{cr}^{\text{max}}$. Notice that the only information that the secondary network requires to dispose from the primary network is the average primary power $|S_p|^2$.

This may be a realistic assumption since usually in spectrum leasing schemes, there are inherently some exchange of information between the primary and secondary networks [6].

There are some cases in which the optimization problem may not be feasible. In these cases, this leasing is not beneficial because there is a trade between primary and secondary networks and primary network has choice to get deal or not. So, if the secondary network cannot satisfy the minimum excess profit for the primary network, the leasing process will not be started.

IV. Numerical Results

The optimization problem is solved by using the Genetic algorithm (GA) optimization toolbox of MATLAB. The form of final optimization problem in (6) is not simple because we have two types of variables with different orders and some terms are in the log function. Genetic Algorithm (GA) is a search heuristic method that mimics the process of natural evolution.

This procedure is routinely used to generate useful optimization solutions and search problems. Genetic algorithms is one of the branches of larger class of evolutionary algorithms (EA), which generate solutions to optimize problems using techniques inspired by natural evolution [10].

Besides, GA is fast and easy to use methods and their results are so trustworthy. So, this method is very helpful for the leasing optimization problem while there are some available tools for solving numerically by genetic algorithm method for practical implementation.

Our numerical analysis showed that the value of estimated parameters does not change significantly by increasing the number of iterations beyond 100 and thus we have set the maximum number of iterations of the GA to 100.

Moreover, the crossover fraction used in GA is equal to 0.7 and the number of primary relays $n_r$ is equal to 3.

We assume that the minimum added rate at the primary receiver due to the leasing process is equal to 0.1 bit/s/Hz (i.e., $\Delta R_p = 0.1$ bit/s/Hz) and the constant value $c$ is equal to 0.3. Based on our simulation results and other related papers, this value give more chances for starting trade between secondary and primary networks.

In practice, this value depends on the strategy of operators about the amount and importance of power consumption in the leasing process. The maximum transmit power of all CR nodes are fixed to 10 dB (i.e. $P_{cr}^{\text{max}} = 10$ dB).

In Fig. 2, we have shown the rates achieved by CR nodes versus the primary power $P_1$ for different number of CR nodes $n_{cr}$. We observe that the CR achievable rates increases with the primary power $P_1$. To better understand this result, we have plotted in Fig. 4 the parameter $\alpha$ versus $P_1$. We observe that $\alpha$ increases
slightly with the increase of $P_1$. We can now understand that the rate increase reported in Fig. 2, is due to the increase of parameter $\alpha$ with respect to $P_1$. In other words, a larger portion of time is dedicated to CR nodes and this increases the rate achieved by the CR network.

From Fig. 3, we observe an increase of the primary achievable rate with respect to $P_1$ in the leasing mode. In other words, we conclude that even when $P_1$ is large, it is still advantageous for the primary network to exploit the leasing framework. We can thus argue that in our considered scenario, the additional rate achieved via leasing for the primary network, is greater than the rate loss due to leasing the time-slot $\alpha$ to the secondary network.

V. Conclusion

In this work, we proposed an optimal beamforming in a framework where the primary network leases some part of its resources to a CR network. Our aim was to find optimal beamforming weightings and the optimal leasing duration so as to ensure benefits for both primary and secondary network.

By using the genetic algorithm for finding the above optimal parameters, we showed that in our approach, both primary and cognitive networks can take advantage from this leasing process in terms of achievable data rates.

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References


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